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The Physics and the Semantics of Quantum Measurement

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In a recent paper, Prugovečki offered a theory of simultaneous measurements based upon an axiomatic description of the measurement act which excludes certain illustrations of simultaneous measurement previously discussed by the present writers. In this article, the fundamental conceptions of state preparation, state determination, and measurement which underlie our research are compared to Prugovečki's interpretations of the analogous constructs in his theory of measurement.

1. PREPARATION AND MEASUREMENT

The confusion deplored in the introduction of the recent article[1] by Prugovečki does not refer so much to the meaning of *simultaneous* measurement as to the definition of *measurement* itself. Sharing his concern for clarity in the rather befuddled domain of quantum measurement, we wish to comment on his attempt to set things straight, an attempt which is noteworthy and somewhat unusual because it takes seriously a number of problems that are ignored by many other writers.

No rational analysis of simultaneous measurements is possible until a precise meaning is agreed upon for the concept of measurement in general. This, it must be stressed, carries a traditional burden which cannot be suddenly unloaded unless there is overwhelming cause for it. Acknowledgment of this degree of prudence is evident even in the writings of Bohr and his followers, who relegate the term measurement to what they call the "classical language." Although we believe that a distinction between a classical and a quantum language is illusory, we take this allegation to imply that the good old meaning of the word should not be tampered with.

Now, in all of science, it seems the term measurement denotes an *operation performed on a system for the purpose of obtaining a numerical value which can, by virtue of the chosen experimental arrangement, be assigned to some definite, nameable observable.* The word "numerical" must be taken in a fairly wide sense, for it may include the values "yes" and "no" (1 and 0) or indeed, if psychological science is included, certain topological values on a j.n.d. scale. But in physics, classical and quantal, restriction to ordinary numerical values having specified units is possible and is in fact practiced.

Quantum mechanics regularizes data within an elegant theoretical format in which the above concept of measurement plays a key role. Three major axioms link the physical ideas of system, observable, preparation, and measurement.

I. With every physical *system*, there is associated a Hilbert space $\mathscr{H}$.

II. Each linear Hermitian operator A on $\mathscr{H}$ corresponds to an observable of the system; any function of A corresponds to that same function of the observable represented by A.

III. With every repeatable empirical method of *preparation* of the system, there is associated a statistical operator ρ, the quantum state; the arithmetic mean $\langle A \rangle$ of a collective of A-data gathered by measurements of A upon an ensemble of systems each prepared (identically) in the manner ρ is given by

$$\langle A \rangle = \mathrm{Tr}(\rho A) \tag{1}$$

It is unfortunate that axiomatizations of quantum theory often either ignore the preparation concept or else refer to it only in a peripheral or tacit manner.

Even worse, the idea of preparation is sometimes construed to be identical with, or to be a form of, measurement. Prugovečki adopts the latter course by interpreting preparation as "preparatory measurement." Thus the quantum state ρ would be produced by a single act of *measurement* (preparatory type). We regard this construction of the preparation act as severely restrictive, unrealistic, and practically useless, and we advocate

instead a broader understanding of state preparation, state determination, and measurement.

While the function of measurement, its role in validating theory, is the same in classical and in quantum physics, its outcomes are quite different. Because of the intrinsic latency[2] of quantum observables, the numerical values of the same observable obtained in measurements of the same type performed on a system in the same state (i.e., having the same statistical operator) may spread, manifesting a statistical dispersion, so that a single measurement cannot be expected to reveal the state. Analogies to this situation may be found in other fields where single measurements are known to scatter. In practice, even in classical physics, precise measurements of one observable yield multiple values which require interpretation based on statistical considerations. On none of these occasions have scientists seen fit to alter the meaning of measurement, or to deprive a single measurement result of significance merely because it could not be repeated. We therefore adhere to the perfectly clear, traditional meaning of measurement, mindful, to be sure, of the fact that a simple measurement can no more determine a quantum state than an accounting (measurement!) of an individual's assets can convey knowledge of the distribution of wealth in a society.

How, then, does one determine a state ? The answer requires a clear understanding of the meaning of *state*. This, at least in physics, is unequivocal. It is the most complete characterization of a system that is attainable and suitable for causal prediction. Since the laws governing different systems are different, states have a variety of definitions. For the simplest dynamical system, the classical mass point, a state is the set of observables (x, p), each a function of time t, and this choice is made because a causal law (Newton's) uniquely controls their evolution. In rigid-body dynamics, a state is given as a larger set of observables, again in order to obtain self-sufficiency in the evolution of states via causal laws. In classical electrodynamics, the state of a field is defined as the set $(\mathbf{E}, \mathbf{B})$, or, alternatively the potentials $(\mathbf{A}, \phi)$, for they are the variables—now functions of space and time—which are propagated by way of Maxwell's equations. What we have termed the causal equation is sometimes called, unfortunately perhaps, the equation of motion.

Returning now to the question of empirical state determination, we note first that in none of the above instances does a simple measurement yield a state, since it produces but the momentary value of a single observable. In the most elementary instance, that of the mass point, at least two measurements are required for a state determination, more in the case of an extended body, an infinite number for the electromagnetic field. This last observation, the need for an infinite number of measurements to determine a state, already suggests that the individual measurement is not the proper road toward state determination, or indeed state preparation.

The difficulty which has just emerged becomes more serious in quantum mechanics. The empirical meaning of the state ρ is given by Eq. (1), which refers to $\langle A \rangle$, a quantity determinable only from an effectively infinite collective of A-data. But even if a particular $\langle A \rangle$ is found, that information is far from sufficient to determine ρ unambiguously. Indeed a *set* of different $\langle A \rangle$'s is needed, the mean values of a *quorum*[3] of observables, the number depending upon the dimensionality of $\mathscr{H}$. Even if it were somehow known in advance that the preparation was refined to the ultimate extent of homogeneity, i.e., $\rho = |\psi\rangle\langle\psi|$, still a doubly infinite set[4] of measurements would be required to find the modulus and phase of the wave function representing ψ.

The foregoing discussion is meant to show why, were we to take Prugovečki's "preparatory measurement" seriously, we would have to reject it because of its artificiality and its inefficacy, not only for quantum mechanics but also for large areas of classical physics. For very simple systems, such as the classical mass point, we concede that it does make sense. The fundamental reason is that in such cases the act of measurement coincides with the act of state preparation, to which we now turn. In general, as will be shown, the two are entirely distinct procedures.

First, we make a trivial point. If state preparation and measurement were identical terms, then states would necessarily and exclusively result from measurements. But many, in fact most, states are presented to us by nature without our cooperation, and it could hardly be said that nature has performed measurements in preparing them. In fact, the primary function of measurement is to identify, as closely as possible, such natural states. But let us shift our attention to deliberate preparation of states and focus it on quantum mechanics.

A common mode of state preparation for an electron consists in the act of sending a stream of electrons down an accelerator tube. By suitable arrangement of apertures and deflection devices, we can, in principle, produce a homogeneous beam (ensemble) with $\rho = |\psi\rangle\langle\psi|$. If the filtration and selection devices are less exacting, or perhaps even omitted, the overall apparatus will in any case prepare some mixed state ρ. At no time do we perform a measurement. It is true, of course, that we could hardly know the prepared state ρ if measurements had not been performed on similarly prepared beams on other occasions. But this is merely to reiterate the truism that all theoretical knowledge is ultimately tested against observations among which measurements form a special class. We wish to stress the fact that in this instance, measurement did not in fact intervene, and that the mysterious "disturbance" with which the Copenhagen school identifies the measurement process surely did not take place.

It seems hardly appropriate to belabor the perfectly clear distinction

between state preparation and measurement, or to present a profusion of obvious examples that exhibit it. However, because the issue—or perhaps only the language—is so beclouded, we offer three more. To place a molecule in a mixture state $\rho \propto e^{-H/kT}$, we heat a gas to temperature T but measure no molecular observable. To prepare the ground state of a hydrogen atom, we merely wait, and know without any physical operation the eventual state vector ψ for the electron. To produce a state we call "a photon of given frequency going in a given direction," we pass a discharge through a sodium gas, focus the light on a slit, and send it through a prism, all of which is preparation; measurement will reveal the D lines.

It should be clear that the preceding considerations prevent us from accepting Prugovečki's identification of "preparation of state" and "preparatory measurement." All measurements are "determinative."

The arguments presented here within a limited context might appear to be purely semantic. Even if this were true, we would regard them as important, for the theory of measurement suffers from linguistic confusion. However, the issue transcends semantics; the distinction we are making becomes extremely significant for an understanding of the quantal measurement process and, in a different way, in the problem of simultaneous measurements, which Prugovečki's article analyzes consistently within the limits of his terminology.

Finally, we should note that state preparation and measurement do occasionally coincide. This is true in many classical situations, where observables are of the possessed variety and their measurements do not scatter appreciably. Thus, by measuring position and velocity of a falling object, one knows, and therefore has prepared, its state. Note, however, that one can also prepare the very same state without measurement by dropping the object from a certain height.

2. QUANTAL TREATMENT OF LEAST COUNT

In Prugovečki's conception of measurement, the result of each individual measurement act is not necessarily an eigenvalue from the spectrum of the measured observable. Instead, he asserts that a single measurement yields a "measured range," an interval which will encompass more than one spectrum point whenever there is a "reading error" for the individual measurement. The familiar operation of measuring position with a meter stick seems to be the paradigm which motivates this unusual view. Thus, if a ruler is scribed in centimeter intervals, the report of a single measurement tells in which of these intervals the measured object was found; similarly, if the least count of the instrument is a millimeter, the position datum for each measurement

becomes more precise in the sense that it locates the object in a smaller interval.

Now, of course, we do not deny that the scale on any measurement apparatus has a least count. However, we must disagree with Prugovečki as to the significance of that fact for the axiomatics of quantum measurement. Rather than heralding the need for an augmentation of the normal postulates of quantum theory, the concept of least count is already implicitly recognized in the established quantal framework.

Consider the physical observable, "X-component of position." The operator X corresponding to this observable is commonly expressed by Dirac's spectral expansion:

$$X = \int_{-\infty}^{\infty} x \mid x > dx < x \tag{2}$$

The choice of an operator to represent position is not the result of any inductive study of rulers; it is a logical consequence of certain philosophical postulates concerning the nature of space. As is evident from (2), the spectrum of the abstract X is *continuous*, but practical rulers, by virtue of their nonzero least counts, can yield for "position" data only the elements of a *discrete* set of indices which label the intervals. Thus it might reasonably be argued that only "measured ranges" can emerge from the application of a meter stick to measure position. Indeed, this intuitive observation seems to be the basis of Prugovečki's measurement theory.

We would resolve the foregoing discrepancy between the characteristics of the theoretic X and the empirical ruler simply by noting that it is a fundamental theorem in quantum mechanics that a measurement of observable A must yield an eigenvalue of A. Hence, if a given measurement scheme lacks the capacity to generate readings which correspond to points in the spectrum of A, that scheme does not constitute an operational definition of A. *Mutatis mutandis* a ruler with nonzero least count is not, strictly speaking, a measurement apparatus for X.

The quantal observable for which a ruler is the correct measuring instrument can be obtained by constructing a function of X which has a discrete spectrum whose elements are numerical labels for the least count intervals of the ruler. In other words, we can readily find, by standard quantum theoretical means, the operator representing the question, "In which least count interval is the object located ?" That operator will be the function of X induced by the function of x defined as follows:

$$f_{\Delta}(x) = a_n\,, \qquad x \in (x_n\,, x_{n+1}) \tag{3}$$

where x_n denotes the position epistemically defined by the nth scale mark on

the ruler, a_n is the numerical label adopted for the least count interval (x_n , x_{n+1}), and $\Delta \equiv (x_{n+1} - x_n)$ is the least count (which is independent of n). Thus the quantal observable measured by a ruler with least count Δ has spectral expansion

$$f_\Delta(X) = \sum_{n=-\infty}^{\infty} a_n \int_{x_n}^{x_{n+1}} | x > dx < x | \tag{4}$$

We conclude that quantum mechanics has no need of the concept "measured range"; for each measurement of $f_\Delta(X)$ yields, in accordance with quantal principles, a *number* a_n from the spectrum of $f_\Delta(X)$. To say that a ruler measures X and yields a "range" a_n with "error" Δ is a semantic mistake. The ruler with least count Δ cannot be used to measure X, but with it, one can measure $f_\Delta(X)$ and obtain eigenvalues a_n . The parameter Δ is an *a priori* property of the ruler and of the operator $f_\Delta(X)$; it is not a "reading error" associated with an individual measurement.

The construct of position represented by X is an idealization which may be understood in empirical terms by regarding its appropriate measuring instrument to be the limit (unattained in practice) of a sequence of rulers generated by letting $\Delta \to 0$.

3. THE POSTMEASUREMENT STATE

Much of the controversy surrounding the quantum theory of measurement revolves around the problem of assigning a state to a physical system subsequent to its measurement. We would prefer in the present context to avoid altogether any detailed analysis of the subtleties of that problem, our views on the matter having been fully set forth elsewhere.[4,5] However, because Prugovečki invokes a postmeasurement state axiom called the "*R*-principle" in arguing against the validity of our conception of simultaneous measurement (cf. Section 4), we are compelled to discuss the question at least to the extent of translating the *R*-principle into the physical language of preparation and measurement.

The axiom under scrutiny is initially stated by Prugovečki as follows[1]:

> *Reproducibility Principle.* If a preparatory measurement $\mathcal{M}_1$ of an observable α prepares a range Δ_1 of values, then any determinative measurement $\mathcal{M}_2$ of α which follows immediately produces the result that the values of α are within a range Δ_2 with $\Delta_1 \cap \Delta_2 \neq \varnothing$. Moreover, for any specific prepared (determined) range Δ_1 (Δ_2), there is an experimental procedure which determines (prepares) a range Δ_2 (Δ_1) contained in Δ_1 (Δ_2).

When expressed without reference to the "measured range" idea disposed

of above, this *R*-principle becomes the old projection postulate, which we reject for several reasons,[1] including its logical falsity (there are measurement schemes for which the statement is untrue) and its pragmatic inutility (no application of quantum mechanics requires such a postulate).

The "modified *R*-principle," which Prugovečki says is the quantal version, is given in these words[(1)]:

> *Modified R-Principle.* If a preparatory measurement prepares a range Δ_1 of values of an observable α, then an immediately following determinative measurement of α will yield a result in Δ_1 with a probability p which depends on the experimental setup and procedure used in the preparatory measurement. For any given interval Δ_1 in $\mathbb{R}'$ containing points of the spectrum of α and any positive number ϵ, there is an experimental preparatory procedure for α such that $p > 1 - \epsilon$.

On first reading, it seems to us that this modified *R*-principle says the following, if we omit reference to the "measured range": If a measurement of A yields a_1, then an immediately following measurement of A will yield a_1 with a probability p which depends upon the nature of the first measurement; for given a_1 and $\epsilon > 0$, there is a first measurement procedure such that $p > 1 - \epsilon$.

However, the statement still resists physical interpretation due to the presence of the *immediate remeasurement* idea. (Indeed, at one point, Prugovečki himself expressed some reservations about that notion.)

Since any measurement operation requires a nonzero interval of time, it is difficult to pin down the precise meaning of immediate remeasurement. If we assume this means that the second measurement process is activated at the instant of completion of the interactions required for the first measurement, then the "modified *R*-principle" really says only this: It is possible to measure A in such a manner that a subsequent measurement will confirm the result of the first measurement; or, more precisely, it is possible to measure A, and to subsequently prepare the system so that the next measurement of A will repeat the first measurement result.

We do not doubt that a dedicated experimenter might contrive an apparatus capable of performing such a succession of measurements and preparations. But that remote possibility is surely unworthy of enshrinement as a "principle." Thus neither the original nor the modified version of the *R*-principle is expressive of a fundamental or universal feature of measurement in quantum physics.

[1] It was the strangeness of the projection postulate which originally motivated philosophical development of the preparation concept.[(6)] See also Refs. 4, 5, and 7.

4. SIMULTANEOUS MEASUREMENTS

Prugovečki rejects an illustration of simultaneous (x, p) measurement given by the present authors[2] on the grounds that the proposed method is incompatible with the R-principle. Inasmuch as we have pointed out above that the R-principle fails to describe exhaustively all conceivable types of measurement acts, naturally we are unconcerned that our (x, p) scheme is omitted from the narrow coverage of that "principle." Nevertheless, in the hope of clarifying the philosophical differences between Prugovečki's approach to measurement theory and ours, we shall briefly reply to several specific points in Prugovečki's critique.

(a) Our scheme for (x, p) measurement at time t is applicable only to systems prepared initially ($t = 0$) such that the position probability distribution vanishes outside a finite interval $(-x_0 , x_0)$. Contrary to Prugovečki's description of this initial condition, the interval $(-x_0 , x_0)$ is not the result of a "preparatory measurement" but is an unmeasured characteristic of the *preparation*. Knowledge of $(-x_0 , x_0)$ is acquired by earlier study of the preparation device involving analysis of X-measurements. No initial X-measurement of any kind is made during a run of the (x, p) measurement process.

(b) The "determinative measurement" of X at time t yields a number x, not a "range" (x_1 , x_2). The arbitrary interval (x_1 , x_2) was introduced in discussing the x distribution at time t for the usual mathematical reason that we cannot define the probability for a single x in a continuous spectrum.

(c) In establishing that our method violates his R-principle, Prugovečki notes that interpolation of an X-measurement prior to t would change the (x, p) result at t. This is true; the proposed (x, p)-measurement scheme requires free evolution of the particle until the terminal X-measurement. But the fact that tampering with an apparatus may foul the measurement does not vitiate the reliability of a measurement procedure when it is properly executed.

(d) Why the (x, p) result at time t should, as Prugovečki asserts, be more naturally regarded as preparatory than determinative escapes us. However, we must emphasize that we have never claimed that the procedure *prepares* the system in (classical) state (x, p)!

(e) Finally, to our surprise, Prugovečki even invokes his R-principle to attack the standard time-of-flight procedure for measuring the momentum observable P at $t = 0$. Yet that method is perhaps the most common operational definition of P. The fact that it violates the R-principle constitutes additional evidence that the R-principle imposes untenable strictures on the measurement concept.

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