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The Concept of Transition in Quantum Mechanics

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The concept of quantum transition is critically examined from the perspective of the modern quantum theory of measurement. Historically rooted in the famous quantum jump of the Old Quantum Theory, the transition idea survives today in experimental jargon due to (1) the notion of uncontrollable disturbance of a system by measurement operations and (2) the wave-packet reduction hypothesis in several forms. Explicit counterexamples to both (1) and (2) are presented in terms of quantum measurement theory. It is concluded that the idea of transition, or quantum jump, can no longer be rationally comprehended within the framework of contemporary physical theory.

1. HISTORICAL DEVELOPMENT OF THE TRANSITION CONCEPT

Quantum physics has evolved during this century through two stages. The first, often called Old Quantum Theory (OQT), was characterized by a prevailing allegiance to the Newtonian–Maxwellian world view, in the sense that the fundamental concepts of the era of mechanism were steadfastly retained, the old laws of nature being subject to amendment, but not to repeal. Consider, for example, the Bohr atom, perhaps the greatest achievement of OQT. Essentially, it was a Rutherford atom, a microcosmic planetary system to be envisaged as a mechanistic entity subject to classical modes of thought, but obeying *amended* classical laws, the amendments being Bohr's postulates. (The electron was still a charged mass point which emitted radiation when accelerated, *unless* it happened to be accelerating in one of the preferred atomic orbits.)

In short, OQT embraced the classical tradition of constructing theoretical models of microsystems in the image of macrosystems, and of framing microlaws in a language depicting more or less *visualizable* behavior for these miniature macrosystems. It was within this philosophical framework that the concept of quantum transition first arose. Thus, in the Bohr atom, an electron was imagined to jump instantaneously from one orbit to another—the famous "quantum jump."

The Old Quantum Theory was, however, rather short-lived. It never coped satisfactorily with the wide range of experimental data that became available during its twenty year existence. And it failed to satisfy certain requirements of reason (the process of amending *classical* laws left them inconsistent), and of scientific epistemology (OQT was becoming so unwieldy that Ockham's Razor could not have been ignored much longer).

The second phase in the development of quantum physics came in the 1920's when many of the lingering concepts of mechanism that had characterized the OQT were renounced in favor of the more abstract modern quantum mechanics, in which microsystems are no longer conceived to be miniature macrosystems, and hence the laws of nature no longer attribute visualizable behavior to physical systems but concentrate instead upon the prediction of measurement statistics.

In view of this theoretical reorientation, it would seem reasonable to conclude that the old concept of transition, rooted as it was in a neoclassical pictorial account of elementary systems, should be regarded as an anachronism, since modern quantum mechanics does not have a conceptual framework which can accommodate "quantum jumps."

Nevertheless, it is not unusual even today to find quantum-theoretical calculations interpreted in terms of those "quantum jumps" of OQT. Suppose, for example, a quantum system prepared in a manner symbolized by state vector ψ is to be subjected to measurements of an observable whose Hermitean operator is A, with eigenvalue equation $A\alpha_k = a_k\alpha_k$. According to the quantal algorithm, the quantity $|\langle\alpha_k, \psi\rangle|^2$ is the *probability that the numerical result of measuring* A *will be the eigenvalue* a_k. But this is not the interpretation many physicists use; instead, $|\langle\alpha_k, \psi\rangle|^2$ is often referred to as the *probability for finding the system* (initially prepared in state ψ) *in state* α_k. Similarly, if ψ has causally evolved from some earlier state ψ_0 without intervention by a measurement act, $|\langle\alpha_k, \psi\rangle|^2$ is often interpreted as the probability that the system has made a transition from state ψ_0 to state α_k.

Some would argue that the distinction between speaking of the probability for the emergence of a number a_k and that for finding the state α_k is unphysical, merely semantic in nature. To support the alleged physical equivalence, it is argued that the act of measurement (1) *disturbs* the measured system, and that, in particular, the disturbance is such that the measurement act (2) *projects* the state of the measured system into the eigenstate α_k corresponding to the measurement result a_k. Hence, according to this view, it is permissible to interpret $|\langle\alpha_k, \psi\rangle|^2$ as the probability for finding the system in state α_k.

It is my contention that the distinction in question is not merely semantic, but *has genuine physical significance.* To establish this claim, counterexamples—constructed within the framework of modern quantum mechanics—are presented below against propositions (1) and (2).

While it is factually correct that measurement operations upon microphysical systems tend to have catastrophic effects upon their states, the notion of uncontrollable *disturbance* of a state by a measurement act, popularized by historic *gedanken* experiments dating back to the turbulent period of transition to modern quantum physics, should not be regarded as a *universal* trait of the measurement act. In Section 3, a measurement interaction will be described formally which leaves the measured system in the same quantum state in which it was initially prepared.

The concept of projection, or wave-packet reduction, is the modern reincarnation of the old quantum transitions. Margenau[1] long ago explored the unscientific subjective origins of this notion; however, there are physicists, most notably Landé,[2] who do reject the subjective interpretation of quantum physics but nevertheless retain the concept of projection as a universal feature of the measurement act. Furthermore, many quantum theorists accept the following compromise[3] on the issue of projection: Interaction between a system and a measurement apparatus generally converts an initial pure system state ψ into a mixture. Hence, if every postmeasurement mixture is postulated to consist of the states α_k with weights $|\langle\alpha_k, \psi\rangle|^2$, then a later selection of subensembles will result in the production of the very postmeasurement states demanded by the projection concept. The conclusion would then be that the aforementioned interpretations of $|\langle\alpha_k, \psi\rangle|^2$ are physically equivalent. In Section 4, a measurement interaction is described which is in no sense whatever projective and which, unlike the counterexample of Section 3, even involves a known physical interaction (spin–spin).

In addition to the philosophical value of exorcising the old notion of quantum transition from modern physics, the insights provided by these considerations have consequences of purely physical interest. For example, abandonment of the ideas of *disturbance* and *projection* would have an impact upon attempts to develop a quantum theory of successive measurements, which does not at present exist. Moreover, it should be noted that the theory of symmetry in quantum mechanics, including the concept of superselection, is founded upon the interpretation of $|\langle\alpha_k, \psi\rangle|^2$. The implications of the conclusions of the present paper for these branches of quantum theory will be explored in subsequent publications.

2. CONCEPTS IN THE THEORY OF MEASUREMENT

Before describing in detail examples of nondisturbing and nonprojective measurement procedures, it seems appropriate to digress briefly to define precisely what I understand the concept *measurement* to entail. Measurement operations are, quite simply, empirical procedures which, when performed upon physical systems, yield the *numbers* called data.

For the data to be of scientific interest, both the mode of preparation of the physical system and the measurement procedure must be reproducible. In the language of quantum physics, a reproducible preparation scheme is represented by a state vector (or, in general, density operator). A measurement procedure is classified by naming the observable (Hermitean operator) about which the procedure is supposed to produce data.

Since microsystems are of greatest interest in quantum physics, it is generally impossible to equate measurement with simple observation of the system itself. Instead, the act of measurement necessarily involves interaction with a secondary system, the apparatus, which in turn produces some effect that can be directly apprehended by the senses. Clearly, not just any interaction will suffice if a measurement operation is contemplated. A measurement interaction must correlate the numerical results which will be obtained from examining the apparatus with the (fictitious) measurement results that would be obtained if the system could be directly observed.[4]

In a quantum-theoretical context, this correlation is achieved by an analysis that may be called *probability matching*. If the interaction of the system and apparatus leaves the composite in a state such that the probability distribution for measurement results of some apparatus observable B after the interaction matches that for measurement results of a system observable A at the onset of the interaction, then the interaction in question has established a correlation of precisely the kind needed to devise a procedure for measuring A. To be explicit, an operational definition of A would read as follows: At time t_0, let system and apparatus begin to interact. When the correlation is established, measure B, obtaining result b_k, which is correlated by the probability matching to A-eigenvalue a_k. The number a_k which has emerged from this procedure is identified as the result of measuring A at time t_0.

In a previous publication[5] by Margenau and the present writer, a classification scheme for measurement procedures was set forth which will prove useful below. The distinction to be drawn among various measurement interactions relates to the breadth of their applicability. A *simple* measurement procedure is one that will succeed in establishing the required correlations regardless of the initial state of the system (or at least for a wide class of initial state preparations). This type of measurement operation is to be contrasted with *historical* measurement procedures, in which the interaction establishes the desired correlation only for a particular initial state; i.e., the measurement is designed using some knowledge of the previous history of the system to be measured.

In terms of the quantal formalism, the evolution operator describing a *simple* measurement interaction is independent of the initial state of the system, whereas that describing an *historical* measurement interaction depends on the initial system state. In the paper[5] where these ideas were first employed, several historical procedures for the simultaneous measurement of noncommuting observables are described. The nondisturbing measurement to be discussed in Section 3 is also of the historical type. The nonprojective procedure considered in Section 4 is a simple quantum measurement operation.

3. A NONDISTURBING QUANTUM MEASUREMENT

Since the primary purpose of the remainder of this paper is to provide counterexamples, within the framework of modern quantum mechanics, which contradict the bases of the transition concept, it is sufficient and indeed desirable to consider only the simplest nontrival physical systems. Accordingly, the present analysis will be limited to the interaction between two distinguishable "spins," one playing the role of system **S**, the other of measurement apparatus **M**. Mathematically, each spin is characterized by a two-dimensional Hilbert space, and the observables for each spin consist of all linear combinations of Pauli spin operators ($\sigma_x, \sigma_y, \sigma_z$) and the identity (1). The combined $\mathbf{S} + \mathbf{M}$ system is then characterized by the four-dimensional tensor product space, and the observables for the composite system consist of all linear combinations of the 16 direct products of $(1, \boldsymbol{\sigma})$ for **M** with $(1, \boldsymbol{\sigma})$ for **S**.

Let α, β denote the eigenvectors of σ_z with eigenvalues $+1, -1$, respectively. The following basis vectors will be used for the composite Hilbert space:

$$\psi_1 \equiv \alpha\alpha, \qquad \psi_2 \equiv \alpha\beta, \qquad \psi_3 \equiv \beta\alpha, \qquad \psi_4 \equiv \beta\beta$$

where, for example, $\alpha\beta$ signifies the tensor product of the α in the **S**-space with the β in the **M**-space. Similarly, in direct products of **S** and **M** operators, the first factor will always be understood to refer ro **S**, the second to **M**.

To construct a measurement procedure, we conventionally adopt some initial state for the apparatus, then seek a correlation-producing interaction that converts the apparatus **M** into a new state which embodies information about the initial state of the system **S**. Specifically, we take α as the initial state for **M**.

To devise a nondisturbing measurement scheme, a unitary evolution operator T must be found that effects the following state evolution for $\mathbf{S} + \mathbf{M}$:

$$T\psi\alpha = \psi\psi \tag{1}$$

where ψ is the initial state of **S**. Such an interaction, if it exists, transfers the state specification of **S** to **M**, yet **S** emerges in the same state it was in at the beginning of the measurement. Hence, measurements upon **M** yield measurement results for **S** without changing the state of **S**.

It is convenient to divide the question as to the existence of T into two parts:

(s) Is there a T independent of ψ which satisfies (1), i.e., can a *simple* nondisturbing measurement be performed?

(h) Can a T be found for any specific ψ which satisfies (1), i.e., can an *historical* nondisturbing measurement be performed?

The answer to (s) turns out to be negative, as the following argument demonstrates. Let $a \equiv \langle\alpha, \psi\rangle$, $b \equiv \langle\beta, \psi\rangle$, so that $\psi = a\alpha + b\beta$. A *simple* nondisturbing T must satisfy

$$T(a\alpha + b\beta)\alpha = (a\alpha + b\beta)(a\alpha + b\beta), \qquad \textit{for every} \quad a, b, |a|^2 + |b|^2 = 1 \tag{2}$$

Invoking the linearity of T and expanding, we may rewrite (2) as

$$aT\alpha\alpha + bT\beta\alpha = a^2\alpha\alpha + ba\beta\alpha + ab\alpha\beta + b^2\beta\beta, \qquad \text{for every} \quad a, b, |a|^2 + |b|^2 = 1 \tag{3}$$

But (3) implies that T must depend on a and b, hence on ψ. To see this, consider the scalar product of (3) with $\alpha\alpha$:

$$a\langle\alpha\alpha, T\alpha\alpha\rangle + b\langle\alpha\alpha, T\beta\alpha\rangle = a^2, \qquad \text{for every} \quad a, b, |a|^2 + |b|^2 = 1 \tag{4}$$

Some matrix elements of T may be found by exploiting the arbitrariness of a, b.

$$\text{Let} \quad a = 1, b = 0; \quad \text{hence,} \quad \langle\alpha\alpha, T\alpha\alpha\rangle = 1 \tag{5a}$$

$$\text{Let} \quad a = 0, b = 1; \quad \text{hence,} \quad \langle\alpha\alpha, T\beta\alpha\rangle = 0 \tag{5b}$$

$$\text{Let} \quad a = b = \frac{1}{\sqrt{2}}; \quad \text{hence,} \quad \frac{1}{\sqrt{2}}\langle\alpha\alpha, T\alpha\alpha\rangle + \frac{1}{\sqrt{2}}\langle\alpha\alpha, T\beta\alpha\rangle = \left(\frac{1}{\sqrt{2}}\right)^2 = \frac{1}{2} \tag{5c}$$

Combining (5a–c), we obtain the absurdity $1/\sqrt{2} = 1/2$. We conclude that there exists no *simple* nondisturbing measurement interaction between two "spins."

The question (h), however, does have an affirmative answer. It is possible to devise a nondisturbing measurement procedure of the *historical* type. Specifically, the following Hamiltonian represents an interaction upon which such a measurement scheme may be founded[1]:

$$\begin{aligned} H = -\tfrac{1}{4}[&6(1)(1) - \sqrt{2}\,(1)(\sigma_x + \sigma_z) - \sqrt{2}\,(\sigma_x + \sigma_z)(1) - (\sigma_x)(\sigma_x - \sqrt{2}\,\sigma_y + \sigma_z) \\ &- \sqrt{2}\,(\sigma_y)(\sigma_x - \sigma_z) - (\sigma_z)(\sigma_x + \sqrt{2}\,\sigma_y + \sigma_z)] \end{aligned} \tag{6}$$

To prove this claim, first Schrödinger's equation for $\mathbf{S} + \mathbf{M}$ must be solved for this H. In the present four-dimensional Hilbert space, the method is straightforward, if tedious; the evolution operator is obtained by exponentiation of H:

$$T(t) = e^{-itH} \tag{7}$$

(we take $\hbar = 1$).

It is convenient to work with matrix representations of the quantities involved, using the basis ψ_1, ψ_2, ψ_3, ψ_4 defined above.

With this basis, the H matrix, denoted by (H), is given by

$$(H) = -\frac{1}{4}\begin{pmatrix} 5 - 2\sqrt{2} & -(1+\sqrt{2}) + i\sqrt{2} & -(1+\sqrt{2}) - i\sqrt{2} & -1 \\ -(1+\sqrt{2}) - i\sqrt{2} & 7 & -1 + 2i\sqrt{2} & (1-\sqrt{2}) + i\sqrt{2} \\ -(1+\sqrt{2}) + i\sqrt{2} & -1 - 2i\sqrt{2} & 7 & (1-\sqrt{2}) - i\sqrt{2} \\ -1 & (1-\sqrt{2}) - i\sqrt{2} & (1-\sqrt{2}) + i\sqrt{2} & 5 + 2\sqrt{2} \end{pmatrix} \tag{8}$$

[1] The question whether and how this Hamiltonian is realized practically is not important in this fundamental context.

Its normalized eigenvectors are

$$\frac{1}{2\sqrt{2}}\begin{pmatrix}1+\sqrt{2}\\1\\1\\-1+\sqrt{2}\end{pmatrix}, \quad \frac{1}{2\sqrt{2}}\begin{pmatrix}1-\sqrt{2}\\1\\1\\-1-\sqrt{2}\end{pmatrix}$$
$$\frac{1}{2\sqrt{2}}\begin{pmatrix}1\\-1+i\sqrt{2}\\-1-i\sqrt{2}\\-1\end{pmatrix}, \quad \frac{1}{2\sqrt{2}}\begin{pmatrix}1\\-1-i\sqrt{2}\\-1+i\sqrt{2}\\-1\end{pmatrix} \tag{9}$$

and the corresponding eigenvalues are, respectively, 0, -2, -1, -3. Therefore, in a representation in which (H) is diagonal, the evolution operator's matrix has the following form:

$$(T(t))_{\mathrm{H\ diagonal}} = \begin{pmatrix}1 & 0 & 0 & 0\\0 & e^{i2t} & 0 & 0\\0 & 0 & e^{it} & 0\\0 & 0 & 0 & e^{i3t}\end{pmatrix} \tag{10}$$

To transforms this back to the original representation, we form the matrix of (H) eigenvectors,

$$(U) \equiv \frac{1}{2\sqrt{2}}\begin{pmatrix}1+\sqrt{2} & 1-\sqrt{2} & 1 & 1\\1 & 1 & -1+i\sqrt{2} & -1-i\sqrt{2}\\1 & 1 & -1-i\sqrt{2} & -1+i\sqrt{2}\\-1+\sqrt{2} & -1-\sqrt{2} & -1 & -1\end{pmatrix} \tag{11}$$

and work out the transformation:

$$(T(t))_{\mathrm{original\ representation}} = (U)(T(t))_{H\ \mathrm{diagonal}}\,(U^{\dagger}) \tag{12}$$

For most values of t, the resultant evolution matrix describes an interaction of the more common variety in which an initially pure state of **S** is converted to a mixture. However, if the interaction is cut off at $t = \pi/2$, a remarkable exception to the usual pure-to-mixed conversion process is obtained.

If $t = \pi/2$, then $e^{i2t} = -1$, $e^{it} = i$, $e^{i3t} = -i$, and the expression given by (12) is much less cumbersome than for general t. In the original representation, the result is

$$\left(T\left(t = \frac{\pi}{2}\right)\right) = \begin{pmatrix}1/\sqrt{2} & 1/\sqrt{2} & 0 & 0\\0 & 0 & 1/\sqrt{2} & 1/\sqrt{2}\\1/\sqrt{2} & -1/\sqrt{2} & 0 & 0\\0 & 0 & 1/\sqrt{2} & -1/\sqrt{2}\end{pmatrix} \tag{13}$$

To see the significance of this evolution operator, consider its impact on the initial $\mathbf{S} + \mathbf{M}$ state $\psi\alpha = (a\alpha + b\beta)\alpha$, which is represented by a column vector with components $(a, 0, b, 0)$:

$$\left(T\left(t = \frac{\pi}{2}\right)\right)(\psi\alpha) = \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 & 0 \\ 0 & 0 & 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} & 0 & 0 \\ 0 & 0 & 1/\sqrt{2} & -1/\sqrt{2} \end{pmatrix} \begin{pmatrix} a \\ 0 \\ b \\ 0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} a \\ b \\ a \\ b \end{pmatrix} \tag{14}$$

i.e.,

$$\begin{aligned} T(t = \pi/2)\,\psi\alpha &= (1/\sqrt{2})(a\alpha\alpha + b\alpha\beta + a\beta\alpha + b\beta\beta) \\ &= [(1/\sqrt{2})\alpha + (1/\sqrt{2})\beta](a\alpha + b\beta) \end{aligned} \tag{15}$$

From (15), it is easy to see that, if $a = b = 1/\sqrt{2}$, i.e., $\psi = \delta \equiv (1/\sqrt{2})\alpha + (1/\sqrt{2})\beta$, then the interaction described by H can indeed be employed to construct a nondisturbing measurement.

To be specific, if $\mathbf{S}$ is initially in the state δ, and $\mathbf{M}$ interacts with $\mathbf{S}$ in the manner represented by H for a time interval $\pi/2$, $\mathbf{S}$ will emerge from the interaction in its original state δ, and $\mathbf{M}$ will then be in a state (its δ) correlated with the original state of $\mathbf{S}$ in such a manner that measurements on $\mathbf{M}$ will yield the same probability distributions that analogous measurements on $\mathbf{S}$ would have yielded before the interaction. Thus, we have constructed a measurement procedure which does not disturb the state of the measured system.

In addition to demonstrating that change of state cannot be regarded as a universal feature of the measurement act, the present example also demolishes another quantum myth, namely, that if the value of one member of a noncommuting pair of observables is known, the other cannot be measured without destroying the certain value of the first. In the present instance, δ, the state of $\mathbf{S}$ before and after a measurement, in an eigenstate of σ_x. But the procedure described above is readily adapted to a measurement of σ_z on $\mathbf{S}$—all that need be done is measure σ_z on $\mathbf{M}$ after the interaction. Hence, we have another example of an historical measurement procedure to be added to the list of those considered in the previous paper[5] mentioned in Section 2.

It is also possible to use the same interaction H to provide an example of a simple nonprojective measurement procedure. (Being simple, it is, of course, one that disturbs the state of S, as proved at the beginning of this section.) However, instead of discussing such measurements with the above H as the interaction, it seems preferable to consider a more realistic interaction (Section 4) which also leads to a simple and nonprojective measurement scheme.

4. A SIMPLE NONPROJECTIVE QUANTUM MEASUREMENT

The interaction between $\mathbf{S}$ and $\mathbf{M}$ discussed in the preceding section was strangely complex and artificial; it was, of course, expressly contrived as a formal counterexample to the proposition that state change by the measurement act is a necessary feature of quantum physics. In this section, we present a somewhat more physically reasonable example of a measurement interaction between **S** and **M**, one which does result in a change of state for **S** but not the projective change of orthodox measurement theory. In particular, for the interaction analyzed below, the probability for obtaining a specified eigenvalue from a measurement on **S** is numerically different from the probability of finding **S** after the measurement in the eigenstate belonging to that eigenvalue.

Consider a spin–spin interaction between **S** and **M** described by the Hamiltonian

$$H = g\boldsymbol{\sigma} \cdot \boldsymbol{\sigma} \tag{16}$$

where g is a real number which may be regarded as the strength of the interaction.

Proceeding as in the previous section, we solve Schrödinger's equation for this H by exponentiating to obtain the evolution operator $T(t)$.

In the representation given by the basis $\psi_1, \psi_2, \psi_3, \psi_4$, defined earlier, the (H) matrix is

$$(H) = g \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 2 & 0 \\ 0 & 2 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{17}$$

Its orthonormalized eigenvectors are

$$\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \quad \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}, \quad \begin{pmatrix} 0 \\ 1/\sqrt{2} \\ 1/\sqrt{2} \\ 0 \end{pmatrix}, \quad \begin{pmatrix} 0 \\ 1/\sqrt{2} \\ -1/\sqrt{2} \\ 0 \end{pmatrix} \tag{18}$$

The first three belong to eigenvalue g, the fourth to eigenvalue $-3g$. Thus, in a representation where (H) is diagonal, the evolution matrix has the form

$$(T(t)) = \begin{pmatrix} e^{-itg} & 0 & 0 & 0 \\ 0 & e^{-itg} & 0 & 0 \\ 0 & 0 & e^{-itg} & 0 \\ 0 & 0 & 0 & e^{it3g} \end{pmatrix} \tag{19}$$

The transformation matrix needed to find the representation of $T(t)$ for the original basis is again just the matrix of H eigenvectors,

$$(U) \equiv \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1/\sqrt{2} & 1/\sqrt{2} \\ 0 & 0 & 1/\sqrt{2} & -1/\sqrt{2} \\ 0 & 1 & 0 & 0 \end{pmatrix} \tag{20}$$

thus

$$(T(t))_{\text{original representation}} = (U)(T(t))_{H \text{ diagonal}}\,(U^{\dagger}) \tag{21}$$

Evaluation of (21) yields

$$(T(t)) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{1}{2}(1+\mathrm{e}) & \frac{1}{2}(1-\mathrm{e}) & 0 \\ 0 & \frac{1}{2}(1-\mathrm{e}) & \frac{1}{2}(1+\mathrm{e}) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{22}$$

where $\mathrm{e} \equiv \mathrm{e}^{it4g}$.

To devise a measurement procedure based on this spin–spin interaction, we again let the initial state of **M** be α; however, since a simple measurement (in the sense defined earlier) is desired, the initial state of **S** is left arbitrary. The initial state of the composite **S** + **M** is therefore

$$\psi\alpha = (a\alpha + b\beta)\alpha = a\alpha\alpha + b\beta\alpha \tag{23}$$

which may be represented by a column vector with elements $(a, 0, b, 0)$.

If the spin–spin interaction lasts for a time t, the state of **S** + **M** will evolve as follows:

$$(T(t))(\psi\alpha) = \begin{pmatrix} a \\ \frac{1}{2}b(1-\mathrm{e}) \\ \frac{1}{2}b(1+\mathrm{e}) \\ 0 \end{pmatrix} \tag{24}$$

i.e.,

$$T(t)\,\psi\alpha = a\alpha\alpha + \tfrac{1}{2}b(1-\mathrm{e})\,\alpha\beta + \tfrac{1}{2}b(1+\mathrm{e})\,\beta\alpha \tag{25}$$

For most values of t, (25) embodies no correlation between **S** and **M** useful for devising a measurement procedure. Moreover, for most t values, **S** alone is, typically, in a mixture state. However, suppose an experimental arrangement is constructed which permits control of the time interval during which the interaction H is effective. (Roughly speaking, this might be accomplished by "crossing beams" of **S** and **M** and varying the velocity of one or both.) If the interaction lasts only for a time interval $\pi/4g$, then $\mathrm{e} = -1$, and

$$T(t = \pi/4g)\,\psi\alpha = a\alpha\alpha + b\alpha\beta = \alpha(a\alpha + b\beta) = \alpha\psi \tag{26}$$

The interaction[2] described by (26) leaves **M** in a state such that measurements of all **M**-observables at $t = \pi/4g$ yield the same probability distributions that measurements of S-observables at $t = 0$ would have given. Hence, we have a measurement procedure.

However, **S** emerges from this measurement operation in the pure state α, contrary to the dictates of the projection theory, even in its mildest form. In particular, the probability for finding **S** in the state α after the measurement (with this method) of any observable whatever is always unity, and consequently, that for finding **S** in any state other than α is always zero.

[2] Evolution operators which trade **S** and **M** states have been discussed formally by Albertson[(6)] and Fine.[(7)]

But the probability of obtaining $+1$, the eigenvalue to which α belongs, when σ_z is measured, is not unity, but $|a|^2 = |\langle\alpha, \psi\rangle|^2$. Hence, the difference between interpreting $|\langle\alpha, \psi\rangle|^2$ as (1) a probability that an eigenvalue will emerge from a measurement act or (2) as a probability that after the measurement act the system will be found in state α is a physical difference, not a semantic distinction. And only interpretation (1) is physically correct.

We observed earlier that in modern quantum mechanics, when ψ has evolved from an earlier state ψ_0, the quantity $|\langle\alpha, \psi\rangle|^2$ is commonly referred to as the transition probability from state ψ_0 to state α, thus perpetuating the concept of quantum jumps peculiar to the Old Quantum Theory. In view of the above considerations, it is therefore apparent that the notion of quantum transition cannot be included with logical consistency within the modern quantal framework.

In short, the concept of quantum jump is no longer a part of quantum physics.

REFERENCES

1. H. Margenau, *Phil. Sci.* **4**, 337 (1937).
2. A. Landé, *New Foundations of Quantum Mechanics* (Cambridge University Press, New York, 1965).
3. F. London and E. Bauer, *Actualities Scientifiques et Industrielles*, No. 775 (Hermann et Cie, Paris, 1939).
4. J. Park, *Phil. Sci.* **35**, 205–231, 389–411 (1968).
5. J. Park and H. Margenau, *Intern. J. Theoret. Phys.* **1**, 211 (1968).
6. J. Albertson, *Phys. Rev.* **129**, 940 (1963).
7. A. Fine, *Proc. Cambridge Phil. Soc.* **65**, 111 (1969).