Chapter 10

Objectivity in Quantum Mechanics[1]

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1. The Meanings of Objectivity

1.1. Objectivity as Ontological Reality

Objectivity means many things. Some philosophers speak of an objective reality behind perceptible things, as that which in some sense *causes* appearances. They hold that the objective lies beyond human knowledge, beyond all experience but is responsible for it. When questions are raised as to the actual existence of an external world, independent of all knowers, it is *this* idea of objectivity that intrudes itself.

The first view to be considered, then, identifies objectivity with ontological existence, and it is a rather common view among physicists who are not given to philosophical reflection. The wholly unquestioning, of course, are satisfied with the attitude of naive realism which takes what is given in sensation to be objective and real. At this stage, the ideas of objectivity and reality are fused together; the refined considerations which force a separation between them have not arisen.

But not many scientists, let alone quantum physicists, are *naive* realists. For if one seeks the objective, understood as the cause of sensations, in the things that appear in sensation, one's search is at once led beyond appearances, since even the simplest scientific observations show that things are not as they are perceived. Atoms and molecules cannot be perceived directly; and if they are to be regarded as "things" in the same ontological meaning as the things we see, a change in the connotation of that word is required. This is particularly necessary when it is realized that the constituents of the atom, being smaller than a wave length of light, cannot be carriers of color; being subject to the uncertainty principle they cannot always possess determinate positions or sizes; in short, when it is realized that they may not be endowed with sensory qualities at all.

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M. Bunge (ed.), *Delaware Seminar in the Foundations of Physics*

Moreover, even realism made sophisticated by the admission that objectivity does not lie within sensation but resides in transcendental entities like atoms, elementary particles, etc., which occasion sensations, is haunted by the fact that the point of contact between the objective and the sensations it produces, i.e., the act in which the real reveals itself to the knower, is shrouded in subjective mystery. This poses a dilemma which troubles all who equate the objective with the ontologically real: The objective is to be inferred from sensations as their common cause; yet it is doubtful whether sensations have anything in common at all. Nobody can show that the color I see is qualitatively the same sensation as the color seen by you. For example, sensations produced by the same objective state of affairs do differ in people who are colorblind.

These are the obvious difficulties of the view in question. If they can be removed, perhaps by postulating that there is a common element in human perception of the world, objective realism has much to recommend it. For it allows itself to be coupled with an old physiological theory of perception which has a solid background in science, leading to an account such as this.

The objective world, which is beyond all experience, is composed of those entities which the scientist, primarily the physicist, continues to discover. They have objectively real attributes like mass, charge, size, shape, position and velocity. They cause objectively real effects in the form of light and sound which our sense organs can receive. The stimulus then travels, again as a physical impulse, to certain places in our brain where translation into conscious response occurs, and a sensation arises.

Clarity and simplicity favor this theory; wide acceptance almost saves it from criticism. It can even explain why sensations differ. To do this it need merely invoke differences in the physical make-up of percipients. Yet there are two major problems which it cannot resolve; one is philosophical, the other physical.

The philosophical problem is to account for the conversion of the physical stimulus into a conscious response, i.e., the age-old mind-body problem. Its solution requires such extraneous devices as the doctrine of psycho-physical parallelism or the Marxist view that consciousness is a manifestation of matter at a certain level of complexity. At any rate, the passage from physical stimulus to conscious response cannot be regarded as an ordinary example of a cause producing an effect; for these, as commonly understood, always act within conscious experience and cannot link the non-conscious with the conscious, nor that which is outside experience with an item of experience.

The physical problem raised by the view in question has its roots in quantum mechanics. Classical physics permitted us to endow the ultimates of the atomic world with those qualities which carry the accent of objectivity in the world of ordinary experience, the qualities on which all reasonable men agree, e.g., position in space, speed, self-identity, size, and mass. These, however, are exactly the attributes whose assignability to the ultimate constituents of the world quantum mechanics has taught us to doubt. As we shall see later, these attributes are closely related to the measurement act, may indeed be engendered by observation, by perception. And thus one is in danger of affirming that the objective aspects of reality beyond experience are somehow dependent on the subjective choice of what one decides to perceive.

A final difficulty in this thesis which identifies objectivity with ontological reality arises in the historical fact that conceptions of the supposedly real entities inferred from sensation change in time. Yet surely the ontologically real should exhibit a high degree of immutability. At this point, ontological realism may cling to a very general concept, like matter, define it in a rather indefinite way which leaves room for changes, and pronounce it the quintessence of the objective. For a time this may succeed, but there are already indications which render the posture of dialectical materialism unreasonable. They are present in the recognition that very unmaterial species of onta, matterless particles and fields, play essential roles in quantum mechanical theories; it may even be that probability fields are irreducible constituents of the ontologically objective. These leave the concept of matter far behind. Then, if we wish to continue to play the game, we have to face these esoteric features and pronounce them objective — for they are after all the residue of what began as an objective thing. And in doing this we could not be sure that fifty years from now an entirely different objective picture would not confront the scientist.

Clearly, what is needed to make an approach of this sort attractive is an act of faith, a postulate of a non-ontological sort which expresses the conviction that the progress of science converges upon a final limit of "truth". This is indeed the conviction which inspires research in science. On the basis of it one can formally define reality to be the goal of the scientific enterprise. Objectivity then is not given but posed as a problem. (One recalls here NATROP's phrase, Das Wirkliche ist nicht gegeben, es ist aufgegeben.) The kind of objective reality thus designated might be called asymptotic.

About it two things must be said. First, it resides no longer beyond experience in a manner that makes it wholly different from experience; for it stands at the end of all experience. It is ideal inasmuch as it is never within human grasp. Objectivity thus becomes an ideal and cannot be assigned to any present phase of scientific operation. Worse, however, is the fact that we do not know it and cannot talk about it in meaningful terms. We therefore dismiss ontological objectivity from further consideration in this article, since it would not allow us to answer the question whether quantum mechanics, a theory currently known, involves or does not involve objective elements.

At this stage, then, we turn to definitions of the word objective that seek its substance within scientific experience, not beyond or at the invisible end of it.

1.2. Objectivity as Intersubjectivity

Since we are now beginning to rely heavily on an understanding of the word *experience*, it is well to state clearly what is to be meant by it. Its root is, of course, the Latin *experiri*, which has a very wide connotation including feeling, sensing, thinking, indeed practically all modes of awareness. Because of its catholicity of meaning it defies clear logical definition. This should not, however, be taken as an indictment, for very few matters that concern us strongly are capable of explicit definition.

The other meaning of experience, which has come to dominate modern thinking, stems — by strange substitution — from the Greek word "en peira ('in trial')". It denotes outer experience, the contingent perceptions and the data that assail us from without. This meaning equates experienced with empirical. Now it will be evident to everyone conversant with the quantum theory that it cannot get along with only that part of Latin *experiri* which the Greek *en peira* singles out. It must include at least the concepts and relations in terms of which science explains its data, and they are not empirical in the narrow sense. The contraction of the pristine meaning of the word experience occurred as the result of the movement known as British empiricism. To do justice to quantum mechanics, it is necessary to restore its original significance and denote by it *all* phases of awareness. For our purposes, however, we may lose sight of affective and conative experiences and focus attention on those which lead to knowledge as distinct from feelings.

Strictly, all experience is first-person experience, and this is almost by definition and at least in the common understanding of the term, *subjective*. But subjective is presumably the opposite of objective. The problem, then, is how to remove the subjective element from first-person experience, that is to say, how to eliminate those features which originate in the person having the experience. Perhaps this cannot be done; perhaps the very act of perception involves, as KANT believed, ingredients contributed in universal ways by every percipient. If this were true communality of perception among many subjects would not remove the personal, or the human, admixture from experience. And if that admixture represents the subjective, it remains immune against the remedy of communality. The Kantian, therefore, would be unimpressed by the view we are now discussing, the view according to which *objectivity* is *intersubjectivity*. To others the thesis has great appeal, and it is foremost in popularity among the views here surveyed.

According to it individual experience cannot be trusted. Everybody knows about sensory illusions, vivid dreams and hallucinations. Only what is trustworthy is worthy of being called objective. Thus, to single out the objective, one may use a method designed to establish "truth". If witnesses of an occurrence differ in their accounts, common sense seems to compel us to dismiss contradictions as untrue and to retain as anchored in truth those parts of their reports which are common. We need not discuss here the philosophical reasons for this compulsion, nor the justification of the procedure. What matters is that the present criterion of objectivity, i.e., intersubjectivity, adopts a common procedure for establishing the truth of reports for the purposes of guaranteeing objectivity of experience. The basic assumption is that the objective content of one first-person experience coincides with that which others report as having experienced under similar external circumstances.

This method, then, leads to the discernment of objectivity in the world of sensations. But the intersubjectivity argument can likewise be used in the realm of ideas. External happenings, common observations suggest conjectures and speculations with respect to unobservable entities; and, in this passage from facts to ideas, intersubjective report of observations is neither a reliable nor a coercive guide. There are many possible interpretations of objective events in scientific theory; and it is becoming increasingly clear that the so-called method of induction, which has at times been supposed to lead with cogency from facts to theory, is quite inadequate to assure the objectivity of theoretical constructs as abstract as those of quantum mechanics, even when the data satisfy the requirements of intersubjectivity. Hence this requirement needs to be applied to ideas also and separately: scientific theories are correct when scientists agree upon them. In this way, theories and the ideas they convey become objective.

Perhaps it is well to make a distinction between two groups who hold views like that under discussion: α) those who use intersubjectivity only for discriminating between subjective and objective *sensations* and either regard ideas as subjective or invoke other criteria for establishing their objectivity, and β) those who rely upon agreement to assure objectivity in both spheres. Evidently class β is more vulnerable than class α because it is harder to agree upon ideas than upon facts.

Specifically, the difficulty with the intersubjective acceptance of interpretations lies in the assignment of competence, in the weighting of the judgment of those who agree or disagree. Only those who understand a theory can rightly be included among contenders, and their number is surely indefinite. If EINSTEIN had counted noses among those who agreed and disagreed with his conclusions regarding the contraction of moving objects or the renowned twin paradox, the objectivity of his theory, judged on these grounds, would have been extremely low. And it might not be high today in view of all the amateurs and half-baked logicians who claim that his conclusions contradict reason. Again, if the writings of all philosophers who deal with the uncertainty principle are placed into the balance, the physicist's ideas may well be thought subjective.

Another difficulty for the class β protagonist arises from the fact that a theory which he must now regard as objective may lose this quality in time. What is troublesome here is not the fact of change — objectivity as well as truth may change in time — but rather the circumstance that the criterion of intersubjectivity provides no reason at all for understanding why a change occurs. If people alter their views there must be reasons aside from the desire to be in the majority which induce at least some of them to do so. Hence it is clear that intersubjective agreement on ideas cannot play the full role of defining objectivity among theories.

While the case is a little stronger for those in class α, even here troubles appear. There are reports of mass hallucinations; large audiences have been subjected to sense deceptions; magicians feel more secure when performing tricks before crowds than in the company of few. These objections, however, seem trivial when compared with the realization that the criterion in question is often irrelevant: One can, without reference to other people, convince himself that a certain observation was erroneous, i.e., non-objective. One of the authors, for example, on one occasion when occupying an office in the Physics Laboratory of the University of Washington, saw a beautiful white cloud hovering some distance from his window. When he commented on this, nobody contradicted him. But the cloud stayed put for days. In his amazement he looked at the map and discovered, all by himself, that he had been gazing at the snow cap of objective Mt. Rainier. It seems as if objectivity can sometimes be distilled in subtle ways out of the experience of a single person.

This possibility gains further interest when it is realized that discovery, the bursting of objective truth into a single mind, is rarely a collective phenomenon. Surely, whenever possible, others will repeat the experiment that led to the discovery and agree with it. Yet there are instances, like the observation of the birth of a nova, which by their very nature are unique. Here the scientist takes the word of a solitary astronomer tentatively to be true and checks it *against other knowledge*, often first-person knowledge. The criterion of validity here is not intersubjectivity but *theoretical consistency* of a certain kind.

Noting this, we now examine theories of objectivity which, while welcoming whatever guarantees the property of communality can offer, seek objectivity within the context of one person's experience.

1.3. Objectivity as Invariance of Aspect

When a thing is seen, many of its properties depend on the relation between the viewer and the object, on his distance, his perspective, the angle from which he sees the object. Shape is one of these properties, e.g., a circle is an ellipse when seen from an angle. Color, size and position with respect to nearby things are others. Indeed it is hard to find observable properties which are not relative and therefore subjective in this sense. Stability does seem to go with a few attributes, like weight and volume, and in others, situations can easily be created in which invariant aspects reveal themslves. For instance, a circle will never look like an ellipse on frontal view, color will not change in the same specified illumination, size of one object will be invariant when it is observed from a standard distance.

The theory at issue holds that objectivity must be assigned to those properties which are, or can be made, invariant. Something is objectively round if under the same specified conditions it always appears round; it is objectively blue if it appears blue in sunlight, and so on. More needs to be said about this theory when it is applied to ideas and interpretations. For the present we deal again with aspects of immediate experience, where this criterion suffices, and again we call this thesis α.

We have already noted that practically nothing is invariant except under specified conditions. Not even weight and volume, which we singled out as nearly objective without conditions, exhibit this character fully. For they are not invariant when crudely apprehended; e.g., when bodies are weighed or spanned by hand. *Balances* make weight, *volumetric procedures* make volumes invariant. Indeed, all sensations, all direct outer experiences, lack the stability in question unless they are severely restricted in carefully prescribed ways. What we are saying is that invariance, and hence objectivity, is conferred by instrumental procedures. We encounter here the problem of operational definition.

The "temperature" I feel in my fingertip when I place it in a hot bath is a subjective sensation in every sense of that word, since it leads to differing reports from different individuals depending on the recent history of the fingertip. It even changes "subjectively" in my own experience when I transfer my finger from an ice bath to hot water. Invariance is achieved, however, if my finger is replaced by a thermometer; the *measured* temperature is invariant and therefore in this sense objective. The procedure in question is called by philosophers, with some pretense to erudition, an operational definition of temperature. Physicists call it simply a measurement.

Every measurement employs an instrument, and in many cases the measuring device seems to be an extension of, or a refinement upon, our sense organs. The use of instruments for the sake of producing objective knowledge is therefore often regarded as philosophically trivial, since they merely enhance the normal method of acquiring factual knowledge. This attitude, however, is fallacious. A measurement does more than improve the accuracy of our normal senses. It establishes a new item in experience, an item somehow correlated with the subjective quality it expresses but not identical with it.

The temperature measured by means of a thermometer, i.e., the point of coincidence between the top of a mercury column and a scale, is different in every sense from the sensation in my fingertip; the weight registered by a balance differs totally from the weight sensation of holding an object in my hand. The force recorded by a dynamometer is not the kinesthetic awareness of a push or a pull.

We conclude, therefore, first, that according to the invariance criterion no immediate sensation is objective; second, that science uses the procedures of measurement to set up invariant counterparts to variant and hence subjective immediate experiences. Strictly speaking these counterparts are contrived, constructed vis-à-vis the flux of sensations; measurement provides rules of correspondence between constructed invariances and the items of direct perception. According to the present theory (part α) measured quantities and the entities, bodies, or systems to which they refer, can be said to be objective attributes or parts of the universe.

The going is rougher when we examine part β of the invariance thesis, which assumes that *ideas* attain objectivity through invariance. This might mean that a certain concept is encountered as the result of manifold avenues of reasoning. In quantum mechanics the question is often asked whether the probabilities assigned to observable events are objective or not. From the present point of view the answer is affirmative if different considerations, indeed all relevant considerations, lead to the same idea and the same value of the probability. Such, we feel, is the case in quantum mechanics. Note however that this kind of objectivity does not satisfy the ontologist, whose attitude we discussed in section 1.1 and who wants to know whether in some transcendental sense events are free from probabilities, since they either happen or not.

On the other hand, the invariance view, conceived as stability of an inference against all proper modes of reasoning, rules out certain conjectures like the wave-nature of electrons because it is the consequence of one set of arguments and not of another.

The technical meaning of invariance in recent physics, while most important in characterizing theories that are likely to be successful in a general sense, is too specific to have much relevance for the problem of objectivity. It is a property of certain mathematical descriptions of natural processes and leads to relativity in the phenomena described. Thus, according to the special theory of relativity, the four-dimensional metric is invariant with respect to certain transformations, while spatial and temporal relations are not. If this were to be interpreted by saying that the metric is objective but spatial and temporal occurrences are not, few physicists would agree.

Similarly, objectivity in quantum theory has sometimes been identified as invariance relative to complementarity[1]. L. ROSENFELD [*1*] has elaborated this viewpoint as follows: "Physical quantities ... correspond with operators susceptible to an infinity of numerical representations. Each of these representations refers to particular conditions of observation, but the equations connecting the operators are invariant for the canonical transformations which express the passage from one mode of observation to another. These equations represent the objective content of the theory, the objective expression for the quantal laws of nature."

In reply to ROSENFELD, MARIO BUNGE [*2*] has suggested that the terms "absolute" and "objective" are erroneously used as synonyms. We note here, in agreement with BUNGE, that the Rosenfeld objectivity inheres in a rather technical invariance which may lose contact with the central meaning of that concept. In the realm of observations the criterion of invariance serves a useful function. And it makes sense to a certain extent in the world of theory although one can hardly subdue the feeling that invariance of theoretical aspect *alone* is not decisive.

A more tenable version of the thesis that in quantum theory objectivity appears as invariance relative to complementarity has been propounded by MAX BORN [*3*]: "I think the idea of invariant is the clue to a rational concept of reality ...

"The final result of complementary experiments is a set of invariants, characteristic of the entity. The main invariants are called charge, mass (or rather: rest-mass), spin, etc.; and in every instance, when we are able to determine these quantities, we decide we have to do with a definite particle. I maintain that we are justified in regarding these particles as real in a sense not essentially different from the usual meaning of the word."

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[1] A concise statement of BOHR's famous principle is the following: "Evidence obtained under different experimental conditions cannot be comprehended within a single picture, but must be regarded as *complementary* in the sense that only the totality of the phenomena exhausts the possible information about the objects." NIELS BOHR, *Albert Einstein*: *Philosopher-Scientist*, p. 210.

1.4. Objectivity as Scientific Verifiability

Science has evolved a method for determining what it regards as *acceptable judgments* in the face of the evidence available at a given time. These judgments reveal its commitments with respect to truth and reality; under the present heading we equate objectivity with scientific truth.

This truth cannot be absolute because it obviously changes as new evidence appears. Whether physicists fifty years from now still believe in electrons as elementary particles is highly questionable; yet according to the view now under discussion electrons are today part of the objective world. If objectivity means permanence of conception, only the ontological interpretation (1.1) is tolerable, and we have seen how its vagueness, its lack of verifiability, make it unsuitable for scientific use.

The method of verification, too, is not fixed in the nature of things, nor is it unalterably grounded in the mind or brain of the knower. Its components are being written while science is made; success of explanation, correct prediction, survival against the vicissitudes of change are its guidelines, and its product is objectivity. The method is immanent, not transcendental; it makes no appeal to ontological reality yet is never forced to disavow it. Operating wholly within experience it relies upon certain organizing principles exhibited by that experience to define the objective.

These principles were studied in a previous publication [*4*] and will here be only briefly reviewed. First, a distinction is made between noncognitive and cognitive experience, and objectivity is placed within the latter. In it are recognized certain components, each with a character of its own. These are the datal protocol experiences, of which a sensation is typical, displaying a high degree of contingency and coerciveness. Near the opposite pole are ideas, concepts, with the chief peculiarity of having been *constructed* by the experiencer. Protocols are always subjective in the beginning — they are the seen color, the felt temperature, the sensed pull called a force. But by the device of operational definition, i.e., by measurement and more generally by certain rules of correspondence, the subjective protocol experiences are made objective in ways that have already been discussed in 1.3.

Strictly speaking, measurement converts raw data into constructs, items of experience for which man himself is responsible. All observables whose symbols appear in equations are constructs — not sensed qualities — in this understanding; and they are objective by virtue of being invariant. Barring minor discrepancies which are dealt with in the theory of errors, everybody measures the same temperature with a given thermometer, perhaps in contradiction to the verdict of his fingertip. But now we are facing the question whether this degree of invariance, conveyed by the standardizing procedures of measurement, is enough to insure objectivity of all theoretical entities, of all constructs.

Operational definitions are arbitrary. One can measure temperature by a variety of different thermometers and obtain different values, each invariant with respect to its own instrument. Thus arises the question: which is the objective temperature, that registered by an alcohol thermometer, that recorded by a mercury thermometer, or the one read from the ideal gas scale? If science were merely a discipline for making accurate measurements one might call them all objective. But temperature, and all measurable quantities, are meant to be significant of something in a deeper sense. Temperature bears reference presumably to the speed of molecules, and objectivity should also apply to them. Their objectivity, however, can hardly be established by invariance of measurement procedures.

On the other hand, it is clear that certain operational definitions, for instance that based on the ideal gas or the Kelvin scale, have special qualities to recommend them. If, for example, I affirm that temperature is proportional to the mean kinetic energy of the molecules in a body, I must prefer KELVIN'S operational definition to that involving an alcohol thermometer. An important fact comes into view at this point, the fact that theory, scientific law, can often discriminate between measurement procedures and show that some are good and others bad. And this judicial function of theory, according to the thesis under study, must be respected in defining objectivity.

With that understanding we continue our survey of the verifying method. The passage from sensation to quantitative constructs makes the data stable, as we have seen. But it does more; it affords the possibility of reasoning about them. There is not much one can do by way of logic or mathematics about the temperature sensation in one's fingertip, but a great deal about the measured number 90° F. Hence the "rules of correspondence" have a dual purpose, to stabilize and to rationalize. Among the constructs we can reason. But in the beginning they are freely chosen, and the latitude of choice is so great that science would flounder if all constructs that are operationally definable or otherwise stand in correspondence with protocol experiences were equally admissible. Hence there must be regulative and discriminative principles which limit that choice.

A large literature is devoted to these regulating principles. They are vaguely referred to by the metaphor of OCCAM'S razor, by economy of thought, and occasionally (though erroneously) by the inductive method. In reference [*4*] they are outlined as a spectrum of "metaphysical" requirements and discussed under the names of logical fertility, multiple connections, stability, extensibility, causality, simplicity and elegance.

None of these requirements can be satisfied absolutely; each makes its demands in competition with the others, and the seasoned scientist knows somehow when maximal justice has been done to them all.

The metaphysical principles are not alone in deciding whether a conjecture is verified because they do not provide the solid links with observation, with protocol experience on which certified knowledge depends. Hence they are augmented by well known processes of empirical confirmation. Perhaps the word "augmented" is too weak to express the crucial importance of the confirming act, and most scientists would insist on placing it before the regulative principles as the means for verification. The point we make is that both are needed, that one is complementary to the other.

Constructs which satisfy the metaphysical requirements as well as the stringent rules of empirical confirmation are called verifacts, and verifacts are the carriers of objectivity in the domain of theory.

Admittedly, this definition does some violence to the common-sense implications of objectivity, as does almost every other careful definition. It suggests, for example, that concepts like the state of a particle which is given by a wave function, though not directly accessible by protocol experience[1], can nevertheless be objective; that as was already mentioned, objectivity of an entity may cease in time; that interesting mathematical constructs which do not refer to the world of observation lack objectivity. The definition contradicts in particular the literal allusion of the word, which seems to refer to an object. If by object is meant a thing, our version of objectivity is much too generous, for it includes innumerable concepts that are non-material and abstract.

It is our belief that the accounts labelled 1.3 and 1.4, invariance and verifiability, are the most defensible on philosophic grounds and at the same time closest to the understanding of objectivity among physicists who work in the quantum theory.

1.5. Intersubjective Subjectivity in Science

The title of this section is not a contradiction in terms. It has already been explained in section 1.2 that, while communality is invaluable to science, it is by no means sufficient to guarantee objectivity. Moreover, although the physicists whose ideas are about to be presented do speak of subjectivity in science, none would deny the simultaneous presence of intersubjectivity.

An examination of philosophic views which attribute to the quantum theory a subjective aspect reveals that the term *subjectivity* is used ambiguously — sometimes in a single paper. The two dominant meanings can be distinguished by using the qualifiers probabilistic and Kantian.

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[1] Although its occurrence can be confirmed by measurements.

Probabilistic subjectivity is a concept applicable only to theories which involve probability and statistics. It is rooted in the doctrine that probabilities represent degrees of knowledge. The remaining sections of this article are devoted to the question as to whether quantum mechanics exhibits subjectivity in this sense.

Kantian subjectivity, on the other hand, is a much broader notion, central to the understanding of the scientific method in general. It is the "subjectivity" apparent in statements such as HEISENBERG'S assertion that what is observed by scientists is "not nature in itself but nature exposed to our method of questioning" [*5*]. The words *idealistic* and *a priori* better convey the intended meaning of *subjective* in this Kantian sense. For example, in the terminology of the preceding section, verifacts might be labelled subjective in order to emphasize their constructional aspect; but such a label runs the risk of committing unintentional distortion. Nevertheless, the term *subjective* is sometimes used in the Kantian sense; and this usage entails certain ideas of crucial significance for the philosophy of science, in recognition of which we advert briefly to the writings of BORN and EDDINGTON.

BORN notes, in a recent essay [*6*], that "fundamentally everything is subjective — everything without exception", thereby emphasizing the first-person character of experience (section 1.2). Intersubjectivity is then established but this in itself does not confer objectivity. In his search for objective knowledge within subjective experience, BORN is led to the mathematical constructs in the exact sciences.

"Mathematics", he says, "is just the detection and investigation of structures of thinking which lie hidden in the mathematical symbols ... These are structures of pure thinking." He concludes that in theoretical physics "hidden structures are coordinated to phenomena; these very structures are regarded by the physicist as the objective reality lying behind the subjective phenomena."

In the language of preceding sections, BORN is suggesting that the verifacts of 1.4 transcend their experiential realm and indeed describe the ontological reality of 1.1. The structures in scientific theory are, for BORN, identifiable with the Kantian *Ding an sich.*

Finally, the "selective subjectivism" [*7*] of Sir ARTHUR EDDINGTON seems to exemplify the notion of communal subjectivity. This is not the place to attempt a survey of EDDINGTON'S fascinating scientific epistemology. Suffice it to say that he lays great stress on the *constructional* nature of verifacts. Even though he favors the idea of an objective ontological reality, EDDINGTON, unlike BORN, does not regard the verifacts of science as identifiable with transcendent components of that reality. Perhaps a succinct expression of his central thesis is in this final colorful passage from one of his books [*8*]: "[Scientific knowledge] is knowledge of structural form, and not knowledge of content. All through the physical world runs that unknown content, which must surely be the stuff of our consciousness. ... And, moreover, we have found that where science has progressed the farthest, the mind has but regained from nature that which the mind has put into nature.

"We have found a strange foot-print on the shores of the unknown. We have devised profound theories, one after another, to account for its origin. At last, we have succeeded in reconstructing the creature that made the foot-print. And Lo! it is our own."

2. Probability

Like many recent scientific theories, quantum mechanics operates extensively with probabilities. In the eyes of many, this alone makes it suspect, for are not all probabilities subjective? To answer this question, we review the idea of probability.

It had a humble beginning in men's concern with games of chance. In its first appearance on the mathematical scene it was taken as an index of confidence in the outcome of an event. Quantified as odds, it expressed a person's expectation of some future happening. Confidence and expectation are private matters which may well vary from person to person. Hence it is easily concluded that probabilities are always subjective estimates of likelihood. This result is in keeping with some modern theories of probability, which are sometimes called a priori theories and occasionally even subjective[1]. Their essential starting point is an insight of LAPLACE who defined probability as the ratio of the number of favorable events to the total number of "equipossible" events. To illustrate, the probability of throwing a five with an honest die is $\frac{1}{6}$ because the die has one face marked with a five and six faces altogether. But evidently, this ratio has nothing whatever to do with the outcome of the next throw, or with the outcome of any one throw of the die. For some reason, not wholly clear from this definition, the ratio is a measure of the confidence one ought to have in the occurrence of a five, and one seems justified in using it as a guide in betting. As an index descriptive of the die the ratio is indeed objective, but as a probability it is not; it represents a subjective measure of likelihood insofar as it refers to an actual event or, put differently, a degree of knowledge concerning that event.

If this interpretation is maintained consistently, the probability must change when the event occurs. Thus, when the die is thrown and a

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[1] To quote H. JEFFREYS, *Theory of Probability,* Oxford Press, (1939): "In fact, no 'objective' definition of probability in terms of actual or possible observations, or possible properties of the world, is admissible."

five appears, the probability has changed from $\frac{1}{6}$ to one; in any other outcome it jumps to zero. The ratio, to be sure, has remained the same, but the facts entailed by the probability interpretation have belied its pretension.

Many difficulties beset this subjective view; among the most troublesome is our inability to specify in many instances the number of equipossible events which enter the ratio. What is the probability that an unknown person is a thief? It is natural to conclude that the probability of his being honest is $\frac{1}{2}$, which is clearly absurd. This is not the occasion to comment on the numerous rescue efforts that have been made to save a priori probabilities and their subjective implications. Suffice it to say that most sciences, especially quantum mechanics, take an approach which avoids this kind of subjectivity.

In their version, probability is a relative frequency of events, or the mathematical limit of relative frequencies. To find the probability of a five one throws the die n times, counts the number of times, say n_5, a five appears, and forms the ratio n_5/n. For small n this ratio fluctuates, but as n increases the fluctuations becomes small and rare. Probability thus defined has reference not merely to the die but also to the sequence of throws; it does not change significantly when a further throw is made — subjective jumps to the value 1 or 0 do not take place. In the sense of 1.3 and 1.4 therefore this probability is an objective quality of a series of throws. The price one pays for invariance is the surrender of meaning with respect to single events; for the frequency definition necessarily involves an aggregate of cases and becomes powerless when confronting a unique occurrence. It cannot handle such notions as the probability that the universe shall cease to exist tomorrow since there is no series of observations of which this is an instance, nor can a single event ever serve to determine a relative frequency.

So far, then, it seems as if there were two probability ideas, one partly subjective and one objective, and that only the latter has a place in science. Closer study however reveals an interesting connection between the two. It is, after all, the case that the two *agree numerically*, although they are logically unrelated. To state this seemingly miraculous coincidence is to invoke a law of nature, a proposition which equates a theoretical construct, the a priori ratio of faces, to a measurable quantity, the relative frequency. The logic of the situation is analogous to that which surrounds every theoretical law. Consider, for example, the law of gravitation: the product of the masses divided by the square of a distance is a construct which is logically unrelated to the operational meaning of a force; yet the law asserts that they are equal and measurement bears this out. We have seen earlier that every fully formulated scientific quantity must have a dual reference, once to datal observations and once to theoretical constructs. Probabilities as they are used in science satisfy this rule. They can be measured, objectively determined, by means of the frequency definition; they can be theoretically predicted by — in the simple instance of the die — LAPLACE'S formula.

In general, the theoretical formula for calculating probabilities is more complex than this, even in games of chance. Basically, that formula suffices for computing the chance that three aces will be dealt to one bridge player, although it needs to be used with care. It does not serve, however, to suggest the probability that two electrons in a hydrogen molecule will be found attached to the same atom. A different a priori definition is required here. The squared modulus of a ψ-function is a probability, perhaps subjective if it is interpreted as an expectation of what might be found in a single observation. It yields a formula for computing and predicting, just as did LAPLACE'S ratio. But when $|\psi|^2$ is coupled, through the laws of quantum mechanics, with the outcome of a series of observations on the position of the "particle" represented by ψ, the subjectivity disappears, the concept $|\psi|^2$ and relative frequency merge into a single meaning which is objective according to our accounts in 1.2, 1.3 and 1.4.

Many philosophers and a fair number of physicists evince displeasure at the thought that probabilities should be accepted on a par with measurable physical quantities, like lengths and sizes and masses. They feel that there is a difference which relegates probability to an inferior status, makes it imprecise and untrustworthy. It is therefore said that the use of probabilities must be indicative of incomplete analysis, a sign that something crucial has escaped detection. This is true in certain situations where probabilities are used for scientific convenience, but it is not conditioned by the nature of probability as a scientific concept. As such it is just as clean, secure, and determinate as any other scientific quantity.

The disturbing feature which seems to contradict this remark is the fact that a probability cannot be determined, measured, in a single act. The number n must be large to make the frequency definition applicable. On the face of it, this contrasts with the measurement of an ordinary quantity, like length, whose value can be read once from a scale. The fallacy of this reasoning need hardly be emphasized here, for every experimenter knows about the vagary of observations. If a good measurement is repeated, a different value will emerge practically every time; and a long series of readings will always result in a distribution of values from which, by statistical consideration, a "true" value can be inferred. To achieve this, the theory of probability enters. Thus, to put it bluntly, the concept of probability is even prior to that of a quantity like length. A single measurement fixes neither an ordinary physical quantity nor a probability, and there remains no reason whatever for excluding the latter from the class of ordinary, decent, measurable physical attributes of the world. Nor can it be denied the quality of objectivity, provided it is taken in its full theoretical-empirical context.

To exemplify antipodal interpretations of quantum mechanical probabilities, we now present typical quotations from the sides of subjectivism and objectivism.

Sir JAMES JEANS, famous for his highly subjective characterization of Schrödinger waves as "waves of knowledge", has written [*9*]: "The wave-picture does not show the future following inexorably from the present, but the imperfections of our future knowledge following inexorably from the imperfections of our present knowledge." In fairness, it should be noted that Jeans means communal, not personal, knowledge [*10*], and therefore confers on quantum theory that degree of objectivity discussed in section 1.2.

In spite of the empirical success of quantum theory, physicists of such stature as EINSTEIN and SCHRÖDINGER have pronounced the theory incomplete on the ground that physics is properly concerned with *objective* physical reality. According to K. R. POPPER, their requirement can be met without eliminating probabilities from quantum mechanics, provided probabilities are understood not as subjective measures of knowledge but in the light of that author's propensity interpretation.

This interpretation is fundamentally in accord with our discussion of the relation between a priori probability and relative frequency. POPPER [*11*] takes "as fundamental *the probability of the result of a single experiment*, with respect to this *conditions*, rather than the frequency of results in a sequence of experiments," although an experimental sequence is undeniably required to test a probability statement. "But now the probability statement is not a statement *about* this sequence; it is a statement about certain properties of the experimental conditions, of the experimental set-up." Probabilities are thus said to "*characterize the disposition, or the propensity,* of the experimental arrangement" to yield certain relative frequencies in ensembles.

POPPER believes that his propensity interpretation "takes the mystery out of quantum theory, while leaving probability and indeterminism in it." SCHRÖDINGER'S ψ-function is said to determine "the propensities of the states of the electron". If these propensities are regarded as objective attributes of the electron, even though they are measured statistically, the ψ-function may indeed reasonably be considered descriptive of objective, physical reality. This view does not differ in its philosophic content from the one put forth by one of the present authors [*12*].

3. The Crucial Issues in Quantum Theory

3.1. Von NEUMANN'S Theory of Mixtures

In quantum mechanics objectivity hinges upon the deeper interpretation of the statistical elements of the theory. To resolve the problem of objectivity in quantum mechanics, it is therefore necessary to understand the mathematical description and classification of quantum statistical ensembles.

The statistics of such an ensemble are characterized by a statistical operator ϱ (whose representation is called the density matrix), which is related to observations through the "mean value postulate": If A is the operator corresponding to some observable (also called A), then the expectation value of A is given by the formula

$$\bar{A} = \operatorname{Tr}(\varrho A). \tag{1}$$

VON NEUMANN [*13*] has shown that the axiomatic correlation of observables to operators, together with general principles of statistics, permits the rigorous classification of quantum mechanical ensembles into two types: pure and mixed. By definition, an ensemble is pure if no subdivision thereof into two subensembles with *different* statistical operators is possible; a mixed ensemble is simply one that is not pure.

In terms of ϱ, the necessary and sufficient condition for a pure ensemble is that

$$\varrho = P_\psi$$

where P_ψ is the projection operator into the closed linear span of the vector ψ. The state of a pure ensemble is thus completely described by a single vector, the famous state vector ψ of quantum mechanics. Accordingly, for a pure case, equation (1) becomes [for $(\psi, \psi) = 1$]

$$\bar{A} = \operatorname{Tr}(P_\psi A) = (\psi, A\psi), \tag{2}$$

the usual statement of the mean value postulate in the quantum theory of pure cases. Equation (2) can be inverted to give the probability w_n that a measurement of A yields the eigenvalue a_n,

$$w_n = |(\psi_n, \psi)|^2, \tag{3}$$

where ψ_n denotes the eigenstate belonging to a_n. (We are assuming eigenvalues to be discrete and non-degenerate throughout this discussion.) Of special importance here is the fact these probabilities w_n, since they are associated with *pure* ensembles, represent maximal precision in quantum theory; they are therefore called irreducible probabilities.

The mixed ensemble, on the other hand, by its very definition, admits of subdivision into component pure subensembles, and will therefore require for its description not only irreducible probabilities but also probabilities analogous to those in classical physics. For definiteness, consider the mixed statistical operator

$$\varrho = \sum_n W_n P_{\varphi n},$$

The mean value postulate gives

$$\bar{A} = \mathrm{Tr}\left(\sum_n W_n P_{\varphi n} A\right) = \sum_n W_n (\varphi_n, A\,\varphi_n). \tag{4}$$

Equation (4) suggests the interpretation that a mixture state is to be regarded as a set of pure states φ_n with respective weights W_n. Thus the mixture state involves just that measure of ignorance which requires the use of probabilities in classical physics.

In statistical mechanics, for instance, one supposes that each molecule is in a definite dynamic state (q_i, p_i), only one does not know nor does it matter which particular molecule occupies that state. It is therefore necessary and proper to introduce the relative number of molecules, W_i, which partake of the state (q_i, p_i). On the basis of this mixture of knowledge and ignorance all theorems of statistical mechanics can be established.

In quantum mechanics, as we have seen, ψ takes the place of the classical state (q_i, p_i). A mixture assigns to every ψ a relative frequency of occurrence W_i, and if all W_i are known the theorems of quantum statistics can be proved.

Now the W_i have a special property: they are *reducible* probabilities That is to say, it is possible by selection, refinement of observation, or some other physical contrivance, to change these probabilities from whatever value they have to 1 or 0. As explained above, this is not possible for the *irreducible* probabilities that inhere in ψ.

In what sense are these probabilities to be conceived? The answer to this question holds the clue to the problem of objectivity in quantum mechanics.

Involved here is the philosophical and to some extent mathematical interpretation of the measurement act. Concerning it we first present the customary textbook version, which finds support in some parts of VON NEUMANN's celebrated book.

3.2. Orthodox Theory of Measurement

The state of a physical system, characterized in general by a statistical operator ϱ, is capable of changing in two entirely different ways. One is the smooth temporal development which for pure cases is in accord with the Schrödinger equation. In general, the change is represented by the evolution operator $T(t_2, t_1)$ such that

$$\varrho_{t_2} = T(t_2, t_1)\,\varrho_{t_1}\,T(t_1, t_2). \tag{5}$$

This type of change involves a special form of causality, suitably named statistical causality, which allows the prediction (or retrodiction) of a probability distribution at a time t_2 when the distribution at time t_1 is given. It does not guarantee dynamical causality, the prediction of a single event at t_2 when a small set of dynamical variables at t_1 is known.

The second type of change in ϱ is said to occur when a measurement is made. It is abrupt, indeed it occurs for all practical purposes instantaneously and cannot be predicted; it has even been called an acausal jump. The orthodox theoretical statement of this change is known as the projection postulate[1], which asserts that after a measurement of A on a single system has yielded the eigenvalue a_n, that system is then in the corresponding eigenstate ψ_n. By equation (1), the measurement of A on an ensemble of systems with statistical operator ϱ leads to the expectation value $\bar{A} = \mathrm{Tr}(\varrho A)$. $\bar{A}$ may be expressed in this way:

$$\bar{A} = \sum_{n,m} (\psi_m, \varrho\psi_n)(\psi_n, A\psi_m) = \sum_{n} (\psi_m, \varrho\psi_m)\, a_m. \tag{6}$$

In the light of the projection postulate, equation (6) may be interpreted by supposing that after the measurement the fraction $(\psi_m, \varrho\psi_m)$ of the ensemble is in the state ψ_m, hence that the total ensemble is then characterized by the statistical operator

$$\hat{\varrho} = \sum_{m} (\psi_m, \varrho\psi_m)\, P_{\psi_m} = \sum_{m} P_{\psi_m} \varrho\, P_{\psi_m}. \tag{7}$$

Equation (7) is the measurement intervention transformation, the second kind of change in ϱ. Of particular interest is the effect of this transformation on a pure statistical operator $\varrho = P_\psi$:

$$\hat{\varrho} = \sum_{n} (\psi_n, P_\psi \psi_n)\, P_{\psi_n} = \sum_{n} |(\psi_n, \psi)|^2 P_{\psi_n} = \sum_{n} w_n P_{\psi_n}, \tag{8}$$

which means that an ensemble consisting initially of systems in the state ψ will be correctly described after measurement as a mixture of the ψ_n with respective weights

$$W_n = w_n \equiv |(\psi_n, \psi)|^2.$$

Having set forth the requisite background material, we now review the analysis of the problem of objectivity in quantum theory as given by the protagonists of the Copenhagen interpretation.

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[1] Because of the mathematical form it takes in SCHRÖDINGER'S wave mechanics, the transformation envisioned in the projection postulate is often called the "reduction of the wave packet."

3.3. Copenhagen Views on Objectivity

HEISENBERG'S philosophic discourses on the Copenhagen interpretation of quantum theory [*14, 15*] note the presence in that theory of a suitably qualified subjectivity — qualified in the sense that any inference that quantum mechanical systems incorporate the mind of the observer is explicitly rejected. Though motivated by the broader Kantian subjectivity (cf. 1.5), Heisenberg bases his main argument that quantum theory is partly subjective upon the statistical elements of the theory; he is therefore primarily concerned with probabilistic subjectivity (cf. 1.5). In the sequel, the term *subjectivity* will refer only to this second meaning.

If subjectivity in quantum mechanics is to arise somehow in statistics, then classical Gibbsian statistics should similarly exemplify a subjective aspect of physics. Far from denying this assertion, Heisenberg builds upon it, observing that the probability function which characterizes the canonical ensemble assigns finite weights to all energies, even though the actual system under study has just one energy. Hence, "the canonical ensemble contains statements not only about the system itself but also about the observer's incomplete knowledge of the system" [*16*]. This two-fold referent for probabilistic statements is the central theme in HEISENBERG'S arguments for subjectivity, both classical and quantum.

HEISENBERG acknowledges the difference in character between the probabilities of the classical canonical ensemble and the quantum pure case when he assigns complete objectivity to the latter. It is rather the manipulable probabilities that arise in quantum theoretical mixtures which Heisenberg regards as partly subjective because, in addition to objective statements about possible measurements, they contain "statements about our knowledge of the system, which of course are subjective in so far as they may be different for different observers" [*17*].

HEISENBERG'S belief that quantum theory exhibits immutably some subjectivity seems to be founded on two propositions.

I. Reducible probabilities, being measures of knowledge which undergo sudden transformations whenever new facts become known, have a partially subjective nature. Hence mixture states are partly subjective.

II. The actual phenomena which are the objects of physical inquiry must necessarily be described as mixtures.

The background for proposition I has been discussed. Proposition II has its origin in certain features of the Copenhagen interpretation of BOHR [*18*] and HEISENBERG [*19*].

The latter insist that, inasmuch as the concepts of classical physics are employed in the description of actual experiments, the proper conception of a physical phenomenon must be so broadened as to embrace not only the behavior of the system S under study but that of the measuring instrument M as well. In the writings of both BOHR and HEISENBERG are found indications that it is the world view of classical physics which is correctly regarded as "real". This seems to be the root of BOHR's complementarity and of HEISENBERG's intriguing Aristotelian pronouncements that quantum states allude only to the "possible", whereas measurement induces a "transition from the possible to the actual" [*20*]. In particular, both advocate the principle that no contrivance may be regarded as a *measuring* apparatus unless its use involves an interaction of the compound system $S+M$ with the classical macroscopic world. Since it can be shown that after two originally independent systems interact, either system considered singly can only be left in a mixture state, it follows from these premises that a physical phenomenon must necessarily be described as a mixture, which is just proposition II.

Incidentally, the meaning of HEISENBERG's assertion that the state of a closed system is objective but not real [*21*], is now clear; for the denial of interaction with the classical world prohibits the "transition from the possible to the actual", which is tantamount to a denial of "reality" to the pure case.

The theme of proposition I, that subjectivity arises with the use of reducible probability notions in a theory, has been analyzed in section 2. What was said there in opposition to the subjective interpretation of probability is not affected by the restriction in proposition I to reducible probabilities. However, if the subjective interpretation of probability is accepted, then the position taken by HEISENBERG on the question of objectivity in quantum theory is logically unassailable. Proposition II, as explained above, is essentially a theorem in quantum mechanics; it is indifferent to the question of objectivity unless used in conjunction with proposition I.

At the 1957 Colston symposium, H. J. GROENEWOLD [*22*] argued that careful analysis fails to justify any subjective interpretation for quantum theory. Since his discussion is based upon the orthodox theory of measurement (cf. 3.2) accepted by the Copenhagen school, we briefly outline it, not only to acknowledge an interesting rebuttal to subjectivist views but also to focus more sharply on those features of quantum theory in which the Copenhagen subjectivity resides.

The GROENEWOLD argument consists of an orthodox quantum mechanical analysis of the following abstract experimental schema. Suppose that a given atomic system S is to be studied by means of a space ensemble of replicas of S. A typical sample of S is enclosed in one of those boxes cherished by physicists which shields its contents from the remainder of the universe. Included with S is a set of selected measuring instruments $M_1, M_2, \dots$, which are triggered to perform measurements of observables $A_1, A_2, \dots$ at successive times $t_1, t_2, \dots$. Each such box is equipped with a sequence of shuttered windows behind which the measured eigenvalues appear. An observer gains cognizance of the measured results by opening the shutters. Whatever he learns by so doing may be incorporated into the statistical operator which describes S. Surely only these latter procedures could conceivably be construed as subjective in any sense. For it must be assumed that the capricious peeking of the experimenter can have no effect on the contents of the box. This very plausible assumption, however, is contradictory to Bohr's stipulation that an essential characteristic of a measuring device is its irremovable interaction with the external world. In the light of Groenewold's schema, such a demand seems neither more nor less relevant that it would be in classical physics. The effect of observation on the pointer behind the window in this case is surely no more significant in a physical sense than, say, the effect of illumination on a falling body. Nonetheless, the interactions of S with $M_1, M_2, \dots$ in the measurement act imply that S must be described as a mixture state. Thus, even with the present modification of basis for proposition II, the locus of Heisenberg's subjectivity — the mixture state — remains firmly embedded in the quantum mechanical description.

Now, looking into the windows on the boxes which house the ensemble will furnish information which permits selection of a subensemble, all the samples of which yielded, say, the same eigenvalue, a_k, of A upon measurement. This act of selection of subensembles is represented mathematically by a "reading" transformation which isolates whatever subjectivity, if any, there may be in this theory:

$$\hat{\varrho}_k = P_{\psi_k} \varrho P_{\psi_k} \tag{9}$$

where $\hat{\varrho}_k$ is the statistical operator of the k-th subensemble immediately after measurement[6]. Equation (9) is based on the principle of the reduction of the wave packet, as may be seen by considering $\varrho = P_\psi$:

$$\hat{\varrho}_k = P_{\psi_k} P_\psi P_{\psi_k} = |(\psi_k, \psi)|^2 P_{\psi_k}$$

or when normalized, $\hat{\varrho}_k = P_{\psi_k}$, an explicit mathematical statement of the projection postulate.

It is at least conceivable that transformations of this type, which do indeed seem to represent sudden changes in quantum statistical description engendered by increase in knowledge, might justify the claim of subjectivity in quantum theory. For the present outline it is enough to assume measurements at two times t_1, t_2, or A_1, A_2, respectively, where A_1, A_2 have simple discrete spectra. All that quantum mechanics can be expected to predict is summarized in $p_{a_1 a_2}$, which is the probability that the measured values of A_1 and A_2 were a_1 and a_2, respectively.

Suppose that between t_1 and t_2, the experimenter notes the relative frequency of the occurrence of a particular a_1 among the readings of A_1. Having thus selected an a_1'-subensemble, he "instantly, acausally, subjectively" alters ϱ by the "reading" transformation, then uses the new ϱ to predict the conditional probability that a_2' will be measured at t_2 in the a_1'-subensemble.

Let ϱ_{t_0} be the initial statistical operator. The procedure outlined above is expressed mathematically as follows:

(1) Time development to t_1:

$$\varrho_{t_1} = T(t_1, t_0)\, \varrho_{t_0}\, T(t_0, t_1)$$

(2) Measurement of A_1:

$$\hat{\varrho}_{t_1} = \sum_{a_1} P_{\psi_{a_1}}\, \varrho_{t_1}\, P_{\psi_{a_1}}.$$

(3) "Subjective act of selection":

$$\hat{\varrho}_{t_1 a_1'} = P_{\psi_{a_1'}}\, \varrho_{t_1}\, P_{\psi_{a_1'}}.$$ [1]

(4) Time development to t_2:

$$\varrho_{t_2} = T(t_2, t_1)\, \hat{\varrho}_{t_1 a_1'}\, T(t_1, t_2).$$

The desired (relative) probability is now given by

(5) $p_{a_1' a_2'} = \mathrm{Tr}\,(\varrho_{t_2} P_{\psi_{a_2'}})$:

$$= \mathrm{Tr}\left[T(t_2, t_1)\, P_{\psi_{a_1'}}\, T(t_1, t_0)\, \varrho_{t_0}\, T(t_0, t_1)\, P_{\psi_{a_1'}}\, T(t_1, t_2)\, P_{\psi_{a_2'}}\right].$$

To what extent that reading and "subjective" adjustment of ϱ affect the quantum theoretical prediction can now be assayed. This is accomplished by calculating the probability $p_{a_1' a_2'}$ *without* benefit of the reading of a_1 in the course of the experiment. When this is done, there comes the revelation that the result is the same whether the experimenter looks or not. But step (3) can no longer be called "subjective". Thus Groenewold concludes that allegations of subjectivity in quantum theory arising from notions about discontinuous changes in the observer's knowledge are essentially verbal.

3.4. Critique of the Projection Postulate

The projection postulate is fashioned after classical science, where one knows that after a system has been found (measured) to be in a certain state it will be there if one looks again immediately afterward.

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[1] This expression is not normalized.

In quantum mechanics, the situation is not quite so simple. In the first place its systems are so delicate that a measurement may alter their states *un*predictably so that even if a value a_i emerges one cannot be sure that it is left in the state ψ_i — indeed a measurement may destroy the system altogether. Secondly, there emerges a more subtle, theoretical difficulty. If the projection postulate is correct, a single measurement, in yielding a determinate ψ_i, would suffice to create knowledge of an entire probability distribution to which ψ_i is related by equation (2). Thus, quantum mechanics is hardly a normal stochastical theory where, as a general rule, a single observation cannot determine a complete distribution.

At this juncture our account is reminiscent of one of the versions of the probability concept, namely the subjective one. According to it, ψ is a measure of *knowledge* of an experimenter before he performs a measurement. When the measurement has been made, knowledge of the outcome is definite and w has jumped to 1 for the property i actually observed, the state from ψ to ψ_i. Hence the projection postulate in the form stated above conveys the subjective meaning of probabilities. Furthermore, barring theories of hidden variables [*23*], states in the form of ψ-vectors are the last instance of appeal in atomic theories, and if they are truly measures of personal knowledge which fluctuates with incidental evidence quantum mechanics must indeed be regarded as a subjective description of man's experience in the sense of all the versions of objectivity presented in section 1.

There is, however, a more cautious variant of the projection postulate, expressed mathematically by equation (9). It acknowledges on the whole the abrupt changes which that axiom envisions but refuses to consider them acausal. A measurement, it affirms, performs a *selection* of systems from an ensemble, thus generating a subensemble containing a smaller number of systems but all in the same measured state. Accordingly, ψ refers to the original ensemble, ψ_i to the subensemble.

We note that this understanding of the projection postulate restores objectivity. The systems found in the post-measurement ensemble were already there originally; if the subensemble to which ψ_i belongs were included among the totality of systems or, in case there is but one system present, if the observations yielding a_i were included among all observations made, ψ would be the same before and after measurement. The "jump" from ψ to ψ_i does in fact have reference to knowledge, but not merely to personal, subjective knowledge. An objective circumstance, the same for all, corresponds to the change of knowledge which is likewise the same for all observers. The selection is an objective procedure in accordance with 1.2, 1.3, and 1.4.

Thus arises the question: which of the two implications of the projection postulate is correct or, if neither, which is nearer the truth? The "truth" in this instance is furnished by a more elaborate theory of measurement which, perhaps strangely, can, like the orthodox theory of 3.2, also be drawn from VON NEUMANN'S book. This theory assumes only the generally accepted postulates of quantum mechanics. Since it is developed in several places [*24*, *25*], we confine our discussion to its major results.

Suppose that a pure case, ψ, of a given system S is present and that a measurement of A is made. When the interaction is followed through in mathematical detail, it is seen that ψ becomes entangled with the state of the measuring apparatus M so that ψ_S and ψ_M convert themselves into a new state ψ_{SM} defined in the combined Hilbert space of S and M. This conversion takes place in strict obedience to the Schrödinger equation and there is no suggestion of an acausal jump. But if now the statistical operator is computed for the state ψ_{SM} in the Hilbert space of S *alone*, it turns out that this operator is not a projection (pure case), but is given by equation (8), a mixture. Indeed the reducible probability with which state ψ_n appears in the mixture has exactly the value given by equation (3). The measurement intervention transformation of the orthodox theory of measurement is therefore *derived* without using the projection postulate at all.

It is already apparent that the first interpretation of the projection postulate is untenable. Measurement changes the state from ψ to a mixture which cannot be written as a single ψ_i. And that is all it does; moreover, it accomplishes this in (stochastically) causal fashion, without violating the Schrödinger equation. But of even greater interest is the fact that the probabilities which appear in the resulting mixture are *reducible*, i.e., manipulable in physical ways. The process whereby one of them is converted to 1 while all other probabilities take on the value 0 is called a selection. Hence the second interpretation of the projection postulate is not far off the mark. While a measurement, in the widest sense, *need* not effect a selection of systems, all in the measured state, it *can* often be coupled with a procedure that achieves this end. We may therefore accept the second explanation as *permitted* by the formalism of quantum mechanics even though we might wish to grant that *there are measurements which are not selective*.

What matters here, however, is this conclusion. The one basic proposition of quantum mechanics which threatens objectivity in at least one of the forms we have discussed is the projection postulate, and it holds this threat only in its first interpretation. That interpretation is erroneous when analyzed fully. Hence the threat is removed. Quantum states, even though they correspond to probabilities, are objective provided the theories of 1.2, 1.3, or 1.4 are accepted.

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