VISTAS IN PHYSICAL REALITY

Festschrift for Henry Margenau

Edited by Ervin Laszlo
and Emily B. Sellon

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PREFACE

Festschriften, when they are haphazard collections of pieces written by colleagues and well-wishers on the occasion of a major anniversary in the life of a distinguished man, tend to be tedious. One can more profitably go directly to the writings of the celebrant, as well as other, more voluntary publications of his well-wishers. However, the editors wish to claim that this Festschrift is different. This is so first of all because of the almost unique combination of interests and competence of Henry Margenau. He is at once a distinguished physicist, an equally distinguished educator, and a prominent philosopher. These broad areas of his extraordinarily active and fruitful career are each represented in this volume in his honor, and this constitutes the particular interest of the collection. Without limiting themselves to paraphrases or empty compliments, the contributors to this book range over the scope of interest of Margenau's work, and, acknowledging its influence and significance, present their own viewpoints and conclusions. Since they include some of the most distinguished men in science and philosophy today, the privilege of having them speak to some broadly defined common concerns in a single volume is a rare one, for which our thanks must go to Henry Margenau, who inspired the papers.

Although the editors have divided the contents into three sections which correspond to the three principal areas of interest represented, the contributions in all sections manifest a comparable breadth of philosophical vision and scientific sophistication. In Section I, some of the most intriguing problems of contemporary physics are dealt with. Wigner considers both external and internal problems: problems of communication and purpose in a science which has continued to grow and diversify rapidly, and questions of knowledge peculiar to the present state of physics. Cosmology and epistemology are among the most fascinating problems with which physicists find themsevles confronted, and Wigner reflects the best in the scientific tradition when he concludes that they are essential to science: what is to be feared is not that we will run out of them but that we may one day get tired of them.

Both Grünbaum and Törnebohm present original contributions to the cause of defending contemporary physics from the charge of instrumentalism, formalism, and anthropomorphic theory construction. Grünbaum demonstrates that human choice and decision cannot invalidate the physical credentials of the statistical entropy principle of classical mechanics. Törnebohm defends the special theory of relativity from a somewhat analogous charge: that it is a theory about clocks and measuring sticks. He distinguishes between theories that map a part or aspect of the real world (*X*-ological maps) and theories that map the behavior of instruments gathering information about the world (*X*-ometric maps). Törnebohm then shows that special relativity theory can be interpreted as a primarily *X*-ological map.

The controversy between Einstein and Bohr in the interpretation of quantum mechanics is the context of Park's contribution. He shows that the epistemology advanced by Margenau already in the 1930s, and restated in the latency theory of observables in 1954, does not require of quantum measurements to come up with possessed observables in the form of measured observables, yet it also does not require the problematic conceptual burden of duality, complementarity, and subjectivism. Finally, Lindsay raises the intriguing question of whether acoustics is, or ever has been, a true physical science. He argues impressively that acoustics is a valid science, and that it is a technology and an art besides.

Section II explores still wider philosophical vistas. It opens somewhat drastically with Yourgrau's radical denial that there is, or can be, *a* philosophy of science. Instead, Yourgrau wishes to treat philosophy of science as a pluralistic venture, consisting of a philosophy of physics, of mathematics, chemistry, and the other natural and social sciences. Laszlo's paper takes a contrary standpoint. Not a monolithic unity of all sciences is presupposed, but the unity of science with respect to epistemology, ideals, and method. Elaborating concepts offered by Margenau, Laszlo attacks the problem of discerning the nature of scientific progress: one which contemporary philosophy of science is on the whole at a loss to provide with cogent meaning. The exploration of philosophical vistas in science is set forth by Caws. The world of experience is closed in on itself, and all our knowledge of the physically real is found within the circle. Can we break out of it? Caws suggests that the isomorphy of well-defined conceptual systems in the sciences present in the minds of all qualified individuals allows us to to so by arguing for the existence of another, underlying system, which is isomorphic with all of them.

Next, Müller-Markus raises an essentially psychological problem and sheds new light on it by drawing on comparisons and analogies with the structure of physical theory. The problem is that of explaining how physicists come by the insights which present major and minor innovations in physics. The science explaining such creativity is not physics but "protophysics," and Müller-Markus lays down its foundations through detailed analyses of discoveries, such as Planck's of the quantum constant and Leibniz's anticipation of it two centuries earlier.

Bacon shifts the discussion to the level of ethics. He subscribes to Margenau's suggestion that a method like that of science can be fruitfully applied to ethics, appraises Margenau's ethics, and then proceeds to outline his own variant of the hypotheticodeductive approach to the problem of constructing and validating a system of normative ethics. Thayer's contribution furnishes an immediate contrast to rational constructivism in matters of general human concern: he argues that in complex societies such as ours there comes a point at which the functions of incompetence exceed those of competence. Competence (specialization to the extent where one man can perform only one kind of task) is a danger not only in complex societies but also in complex social and cognitive institutions, such as science.

This takes us to the third and last section of this volume. Margenau has been an articulate and long-standing advocate of integration and reform in education, especially in the sciences and the philosophy of science. Section III has these issues as its focus. Here Cassidy reports on his sustained experience with using Margenau's epistemology of science as a "philosophy by example" that offers a frame of reference within which cognitive knowledge, subjective experience, and action in applying these to practice can unify the task of the university and give meaning to education and life. The concluding paper by Sellon brings together some of Margenau's own ideas on integrative education as he has presented them over many decades of writing and teaching, and indicates some of the possibilities and directions toward which it may be tending.

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James L. Park

THE MEASUREMENT ACT IN QUANTUM PHYSICS

Until the advent of modern quantum theory with its profoundly statistical nature, most uncritical scientists probably espoused a doctrine of epistemological realism which presumed, rather boldly, that the mechanistic world described by the classical theories constituted quite literally a model, a facsimile, of ontological reality. With the empirical failures of mechanism and the striking successes of the irreducibly probabilistic quantum theory, the prevailing philosophical inclinations of physical theorists shifted during the 1920s toward epistemological idealism. Unfortunately, the considerable critical merits of the latter, an enlightened approach to science founded by Kant centuries before, were almost lost as physicists of the so-called Copenhagen school extrapolated idealism into solipsism until the orthodox natural philosophy of quantum physics had degenerated to a subjectivistic blend of physics and psychology wherein physical states of physical systems could be altered by the mere deliverance of information into the consciousness of the experimenter.

In dissent against this grotesque form of scientific philosophy, a minority chorus arose which included the voices of such distinguished iconoclasts as Einstein and Schrödinger, who condemned the structure of quantum physics itself as well as its Copenhagen interpretation. It has since become part of the folklore of physics that Bohr, the prophet of

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Copenhagen, outwitted Einstein by adroit dialectical argumentation. I believe that the opposite is true: that Einstein understood the essence and the consequences of quantum theory better than Bohr ever did, that Einstein for his own metaphysical reasons would not accept those consequences, and that Bohr simply smothered Einstein's cogent analysis in that obfuscous fog called Complementarity.

Henry Margenau emerged in the 1930s as a physicist-philosopher who shared neither Bohr's abstruse views nor Einstein's reluctance to accept the new physics. Professor Margenau has since shown us that epistemological idealism does not require physical theory to exhibit subjectivist constructs, that indeed *objectivity*—physical reality—may be identified *within experience.*[1] In the Margenau epistemology, quantum physics may be comprehended with remarkable lucidity, yet without recourse to the ponderous verbiage of wave-particle duality, complementarity, and subjectivism.

In this essay I wish to honor Professor Margenau by offering an analysis* of quantal measurement inspired by his latency theory of observables. In particular, several types of measurement procedures will be described in order to illuminate some of the philosophical issues associated with the quantal measurement act.

The Primitive Measurement M_1

Classical and quantum physics alike focus attention on constructs called physical systems, which serve as the fundamental constituents of the theoretical models in terms of which experience is catalogued. All physical systems are regarded as carriers, in some sense, of observables.

A philosphical analysis which elegantly pinpoints the essential difference between the fundamental models employed in classical and quantum physics has been given by Margenau in his latency theory of observables.[1–3] This theory classifies the conceptual relations which link systems with their observables under two headings, possession and latency. Possessed observables are the hallmark of classical physics, where observables were regarded as labels attached to a system, i.e., as properties of the system. Within such a framework, it did not seem unreasonable to

*The following section of this paper is an abridged version of parts of several articles in *Scientia: 103* (1968), 569; *105* (1970), 269; *105* (1970), 320.

interpret a system with its observables as a pictorialization of the *Ding an Sich* responsible for, but beyond the grasp of, experience.

The characteristic quantal relation of latency is subtler than that of possession. The organized data of physics consist of numbers which emerge from experiments in which the acts called measurement are performed on physical systems. Each observable associated with a system is equipped in principle with operational definitions which give methods for the measurement of that observable. When a measurement of a given observable is performed, the numerical measurement result which emerges is recorded as the value of that observable at the instant defined by the onset of the measurement act. The difference between latency and possession lies in the theoretical interpretation of such numerical data. If possession is the assumed relation between system and observable, the measurement result is taken to be a revelation of which value of the observable the system possessed just prior to the measurement act.

If latency is the assumed relation between system and observable, the measurement result is not given this transcendent interpretation; the system is not regarded as an object bearing definite numerical values for all its observables either before or after measurement. Instead, a report of experimental results is confined to a minimal statement of how the systems were prepared for study, which observables were measured, and what numbers these measurements yielded. Similarly, a theoretical prediction of experimental results can have only the dispositional form "if system S is prepared in the manner Π and measurements of observable A are performed on S, then the numerical result a will emerge with probability $W(a; \Pi)$."

I shall call this minimal account of natural phenomena the *preparation–measurement format* for physical problems to distinguish it from the classical ideal of describing with Cartesian clarity the continuous evolution of an external world. Epistemologically, the latency viewpoint is entirely adequate for the scientific confrontation of experiential data and abstract concepts; but it stops short of identifying the numerical data of science with prepossessed properties of systems. Instead, numerical values of latent observables are considered to emerge from systems only when educed by an act of measurement.

Margenau has stressed for decades that the important concepts of preparation and measurement are *not equivalent.*[(3)] The term "preparation" signifies a reproducible physical act involving the system of interest, whereas the term "measurement" refers to a physical procedure on the prepared system which yields a numerical datum. To gather the statistical information from which the probabilities of quantum physics may be extracted, it is necessary to reprepare in an identical manner the system of interest (or another of the same kind) many times, and to perform a measurement subsequent to each repreparation. The probability $W(a; \Pi)$ associated with the measurement result a of an observable A for a system prepared in the manner Π is operationally defined as the relative frequency with which a emerges from such *ensembles* of identically prepared systems.

Unlike the classical approach to physical phenomena, the preparation-measurement format embodies a certain finality in the measurement act. The impact of measurement on a physical system can range from negligible (macrosystems) to catastrophic (microsystems); sometimes systems are even annihilated. Accordingly, quantum physics cannot include any *universal* proposition concerning the postmeasurement condition of physical systems.

The mathematical structure of quantum theory may be displayed in three simple axioms which employ the language of the preparation-measurement format:

- A1: For every physical system S there is a Hilbert space $\mathcal{H}$. If S_1 and S_2 have Hilbert spaces $\mathcal{H}_1$ and $\mathcal{H}_2$, the Hilbert space of the composite system $S_1 + S_2$ is the tensor product space $\mathcal{H}_1 \otimes \mathcal{H}_2$.
- A2: The linear Hermitean operators $A, B, \ldots$ on $\mathcal{H}$ correspond to physical *observables* of the system S. When a *measurement* $M_1(A)$ of observable A is performed on S, a real number, an A-datum, emerges.
- A3: Every mode of *preparation* Π of a system S is represented by a statistical operator ρ. The arithmetic mean of A-data obtained from measurements $M_1(A)$ on an ensemble of systems S each prepared in the manner Π is given by $\langle A \rangle = Tr(\rho A)$.

Now, when as in classical theoretical physics, system and observable are connected by the primitive relation of possession, the preparation-measurement format seems an overly cumbersome expression of a simpler idea, *viz.*, that the act of measurement is merely a careful observation of the properties borne by the system. If a system can be thought of as carrying numerical labels, one for each observable, measurement becomes nothing more than a procedure for discovering at some instant of interest the values of one or more of these labels.

However, in the mathematical framework set forth above, it is impossible even to conceive theoretically of a preparation Π which would result in certain predictable values for all observables on measurement M_1. This is the essence of von Neumann's proof that there are no dispersionless ensembles, of Heisenberg's uncertainty relations, and it is the logical background for Margenau's latency interpretation.

In quantum physics, experiments remain describable in the preparation-measurement format; but, because of the inevitable quantal uncertainty, no consistent labeling scheme is possible. Consequently, the primitive relation *possession* must be abandoned, and measurement can no longer be interpreted as yielding revelations of preexisting values for observables. The preparation-measurement format itself is in quantum physics a primitive concept. More precisely, quantum theory employs the notion of measurement M_1 in a manner that must be regarded as logically primitive. Thus when a quantum theorist asserts that if a system is prepared in a specified manner subsequent measurements will yield particular numbers with stated probabilities, no deeper, hidden meaning is intended. In particular, no prediction is made about properties of the system (since it possesses none); the prediction is literally (and physically!) about measurement results subsequent to a specified preparation. Hence that which is predicted cannot even be stated without using the notion of measurement; therefore, measurement M_1 is a primitive term.

M_1 is, in fact, the primitive relation that plays the same role in quantum physics that possession did in classical physics. Wherever in classical theory the statement "system S possesses observable A with value a" would occur, the analogous place in quantum theory will feature the dispositional statement "if observable A is measured on S, the number a will emerge (with some specified probability)." Thus a quantum observable is associated with the system but no numerical label is attached to the system; however, when elicited by measurement, a numerical datum will emerge for that latent observable.

Complementarity is not without its contemporary apologists. Among these is Feyerabend,[(4)] who several years ago declared that Margenau's latency theory consisted merely of Bohr's views expressed in "fancy terminology." In rejoinder to this rather snide allegation, I should like to place the quantum philosophies of Bohr and Margenau in historical perspective.

Consider Copernicus, precursor to Kepler, Galileo, and Newton, whose works culminated in the decline and fall of Aristotelian science and its attendant philosophies. Aristotelian physics classified all phenomena into the categories of terrestial and celestial. For terrestial bodies linearity of motion was considered natural, but for celestial bodies circularity of motion was the dogmatic requirement. Thus in his *De Revolutionibus* Copernicus,[5] following Aristotle, asserts that

> . . . the motion of heavenly bodies is circular. Rotation is natural to a sphere and by that very act is its shape expressed.

He next proceeds to review the complexities of planetary movements, the departures from uniformity which would seem to argue against the necessity of circular perfection in the heavens. But then, having exposed the very data which inspired his bold advocacy of heliocentric astronomy, which would later motivate total abandonment of the circularity dogma by others, Copernicus made a remarkably conservative statement:

> Nevertheless, despite these irregularities, we must conclude that the motions of these bodies are ever circular or compounded of circles.

In the present century, the dismal failure of classical physics to account for atomic phenomena has resulted in what might be called the Copernican stage of a developing quantum revolution. At the microscopic level, the classical approach to physical problems has essentially been renounced. Moreover, it is generally acknowledged that quantum physics is in principle applicable to all physical phenomena, large and small. Nevertheless, the prevailing intuition with which even many modern physicists approach macroscopic phenomena is deeply embedded in the empirically discredited fabric of classical physics.

This situation is somewhat reminiscent of the medieval celestial-terrestrial dichotomy. In fact, some of the pronouncements of pioneer quantum theorists exhibit a striking resemblance to the perorations of Copernicus in defense of circularity. For example, Bohr[6] seems to insist on the necessity of the classical outlook for macroscopic experiences:

> . . . however far the phenomena transcend the scope of classical physical explanation, the account of all evidence must be expressed in classical terms.

Similarly, Heisenberg,[7] in explaining the so-called Copenhagen interpretation of quantum theory, ascribes to classical physics a rather sacrosanct position in scientific methodology:

> Any experiment in physics, whether it refers to the phenomena of daily life or to atomic events, is to be described in the terms of classical physics . . . the application of these [classical] concepts is limited by the relations of uncertainty. We must keep in mind this limited range of applicability of the classical concepts while using them, but we cannot and should not try to improve them.

It is my contention that the Copenhagen interpretation of quantum physics, a key point of which is indicated by the preceding quotations, is not the last word, but that it bears the same relationship to the current quantum revolution that the sixteenth-century work of Copernicus bore to the great seventeenth-century scientific revolution. The Copenhagen bifurcation of physical intuition into microscopic and macroscopic components is like the old celestial-terrestrial dichotomy in that it attempts to conventionalize a paradoxical melange of mutually incompatible concepts in order to conserve, in part, the no longer adequate Newtonian *Weltansicht.*

In the latency interpretation of quantum physics, and in Margenau's epistemology generally, no such absoluteness, no inevitability, is attributed to classical constructs; hence there is no "wave–particle duality." And that is a significant distinction between complementarity and latency.

The Operational Measurement M_2

If taken literally, the term "observable" as used in theoretical physics is quite misleading; for if observation denotes immediate apprehension by the senses, then most physical "observables," classical or quantal, are certainly unobservable! Consider, for example, the linear momentum P_x of a system S. Classical mechanics assumes that S has a P_x-value at any time t; quantum mechanics assumes that a P_x-value will emerge if $M_1(P_x)$ is performed at any time t. Yet in neither case is P_x directly observable. To obtain a P_x-value, a procedure is required which is considerably more elaborate than mere observation. Somehow an operation must be devised which establishes a correlation between the sought-after but unobservable P_x-value and another observable D for which the act $M_1(D)$ is a matter of direct apperception.

The creation of such correlative procedures is the art of experimental physics, which entails complex operations of data extraction commonly called measurements. Like any other physical process, measurements in this sense can be described quantally only in terms of the primitive measurements M_1. For example, an empirical measurement scheme for P_x is legitimized theoretically by proving that the procedure relates $M_1(P_x)$ and $M_1(D)$ in such a manner that the D-data actually gathered are in some sense equivalent to, or translatable to, the desired but not directly accessible P_x-data. To distinguish the term "measurement" as a physical operation which correlates observables from the *primitive measurement* construct M_1, let the former be designated M_2, the *operational measurement.*

A familiar illustration of an $M_2(P_x)$ is the time-of-flight method, wherein it is assumed that for the position coordinate X, $M_1(X)$ is either directly performable or an $M_2(X)$ is readily available. If x is the X-datum obtained from $M_1(X)$ at time t and m the mass of the system, then the number mx/t is taken to be equal to the P_x-datum that $M_1(P_x)$ would have yielded at $t = 0$, provided that three conditions are fulfilled: (a) The preparation at $t = 0$ must be one for which X-data at that initial time would display a probability distribution of compact support about $x = 0$; i.e., the probability density for X-data at $t = 0$ must vanish outside a finite neighborhood of $x = 0$. (b) The system must evolve *freely* in the interval from $t = 0$ until the terminal act $M_1(X)$ which yields the datum mx/t. (c) The precision of this "P_x-meter" increases as t increases; thus t must be as large as possible.

It is instructive to compare the classical and quantal measurement (M_2) theories which justify the claim that the foregoing procedure measures P_x. In the classical case it need only be noted that a particle always *possesses* some value for P_x and that in free dynamic evolution that value is constant; hence the final $M_1(X)$ simply enables the experimenter to ascertain which P_x-value the particle had all along its flight.

In the quantal version, on the other hand, it is physically meaningless to assert that the system had any P_x-value at any time during the process. If the initial preparation had been characterized by a statistical operator which assigned unit probability to a particular P_x-value, then to be sure P_x would retain its spiked probability distribution as long as the system remained dynamically free. However, due to the uncertainty relation $\Delta X \Delta P_x \geq \hbar/2$, the above condition (a) for the efficacy of the time-of-flight method would then be violated. Hence preparations for which the time-of-flight $M_2(P_x)$ is valid do not admit of the interpretation that the system possesses an unknown P_x-value.

To be specific, the detailed quantum mechanical analysis[8] of this $M_2(P_x)$ can and does affirm only this proposition: the probability that $M_1(X)$ at time t will yield the number mx/t equals the probability that $M_1(P_x)$ at $t = 0$ would have yielded the same number provided that conditions (a), (b), and (c) are satisfied. The answer, if there be one, to the somewhat ontological question as to whether in a single run of the experiment the value mx/t is truly the same as the datum that $M_1(P_x)$ would have yielded at $t = 0$ is beyond the scope of the probabilistic quantum theory.

Although the time-of-flight $M_2(P_x)$ is only one type among a diverse variety of conceivable operational measurements, our careful if somewhat belabored scrutiny has revealed one structural feature paradigmatic of all *quantum* theories of M_2: a procedure is quantally certified as an $M_2(A)$ if it can be proved that the theoretical probability distribution for A-data *matches* the probability distribution for D-data, where $M_1(D)$ is a directly performable act.[9]

From a classical viewpoint, simple *probability matching* would seem inadequate to justify identification of a given operation as a measurement scheme; nevertheless, it is the best justification that quantum mechanics, with its latent observables and irreducible probabilities, can possibly offer.

Most textbooks discuss the Stern-Gerlach "spin-meter" as the prototype operational measurement. At first glance, the correlations it establishes may seem stronger than simple probability matching. Indeed, after the system has interacted with the inhomogeneous magnetic field, there is a *correlated joint probability distribution* in the variables position and spin such that from a position measurement datum one may infer the value that a simultaneous primitive spin measurement would have yielded. However, the function of the "spin-meter," of which the field is merely a component, is to measure the spin of the system prior to its interaction with the meter. The only sense in which this is achieved is that the probability distribution for spin is unchanged by the interaction, a *probability match* which does not offer any extraphysical guarantee that a preinteraction primitive spin measurement would have yielded the same result as a postinteraction primitive spin measurement in the same run of the experiment. But no such assurance should be expected from an indeterministic theory which predicts only probability distributions, a theory in which observables are fundamentally latent.

To summarize: an empirical scheme is certified to be an operational measurement $M_2(A)$ if the quantum theoretical probability distribution of $M_1(A)$ data matches that for $M_1(D)$ data. Each number on the "D-scale" which emerges from the act $M_1(D)$ may be transformed by means of the $M_2(A)$ theory to that number on the "A-scale" having the same theoretical probability, and as a matter of convention that A-value may be defined as the datum that the unperformed primitive measurement $M_1(A)$ would have yielded. However, it is only the collective of such A-data obtained from an ensemble of many runs that is scientifically important.

Misconceptions Concerning Quantal Measurement

There have been many attempts to characterize the measurement act in quantum mechanics more fully, more explicitly, than does the foregoing discussion of Π, M_1, M_2, and probability matching. Consequently the literature of physics abounds with references to a number of alleged after-effects of measurement, among these the following propositions:

a. Measurement determines the unknown quantum state vector.
b. Measurement destroys coherence.
c. Measurement entails uncontrollable disturbance of the measured system.

We shall demonstrate below that none of these is a universal trait of the operational measurement M_2, hence that all are physical overspecifications of the measurement act which, if regarded as axiomatic, would unreasonably constrict the quantal meaning of the term "measurement."

To focus attention on structure rather than detail and to avoid fruitless tedium, we consider only the simplest nontrivial physical systems. *viz.*, those whose individual Hilbert spaces $\mathcal{H}$ are only two-dimensional. The observables for each such system comprise all Hermitean linear combinations of the identity operator 1 and the Pauli operators σ_x, σ_y, σ_z on $\mathcal{H}$. Let α, β be the normalized eigenvectors of σ_z belonging to eigenvalues $+1$, -1, respectively. If two systems S_1, S_2 are combined to form a single composite system, the Hilbert space $\mathcal{H}_1 \otimes \mathcal{H}_2$ for the latter will be four-dimensional and may be spanned by the orthonormal set $\alpha\alpha$, $\alpha\beta$, $\beta\alpha$, $\beta\beta$, where, for instance, $\beta\alpha$ denotes the tensor product of the β in $\mathcal{H}_1$ with the α in $\mathcal{H}_2$. This notation may be extended in an obvious way to embrace composite systems having N constituent subsystems. Similarly, let the direct product of operators be represented by ordered juxtaposition; for example, $\sigma_x 1$ signifies the direct product of the σ_x on $\mathcal{H}_1$ and the 1 on $\mathcal{H}_2$, where $\sigma_x 1$ itself is of course an operator on $\mathcal{H}_1 \otimes \mathcal{H}_2$.

Does Measurement Determine the State Vector?

Let ρ be the statistical operator for a given preparation of a system S and P_α the projector onto α. All presentations of quantum theory attribute special significance to the number $\mathrm{Tr}(\rho P_\alpha)$, but there are two distinct versions:

i. $\mathrm{Tr}(\rho P_\alpha)$ is the probability that $M_1(\sigma_z)$ or $M_2(\sigma_z)$ will yield the datum $+1$.
ii. $\mathrm{Tr}(\rho P_\alpha)$ is the probability that S will be found in the state α.

The language of the rule (ii) is based on the projection, or wave packet reduction, postulate according to which a measurement, in addition to yielding a number, *prepares* S so that an immediate remeasurement will yield the same number. Thus if σ_z is measured with result $+1$, S then will be in, or "have been found in," the eigenstate α. The irrationality of the projection postulate was exposed long ago by Margenau on both physical and philosophical grounds.[3] Indeed, it is somewhat vexing to find that wave packet reduction has yet to be exorcised from contemporary physics literature.

The pragmatic experimenter, who gathers and records in his log book *numerical* data—not Hilbert vectors—would probably regard (ii) as the theoretician's quaint rendition of (i); thus it might seem that the only difference between the two versions is a semantic one.

To see that the difference is quite physical and that (ii) is in fact erroneous, consider the following M_2 scheme for measuring observables of a system S_1. Let S_1 interact with an auxiliary system S_2 (the "apparatus") for a time interval in which the causal evolution of the composite system is described by the unitary operator T whose matrix representation in the basis $\alpha\alpha$, $\alpha\beta$, $\beta\alpha$, $\beta\beta$ is given by

$$(T) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{1}$$

We have shown elsewhere[10] that such an evolution operator may be generated by a Hamiltonian proportional to $\vec{\sigma}\cdot\vec{\sigma}$; but this kind of detail is unimportant in the present analysis. It is always possible in principle to find a Hamiltonian H and a time interval τ such that any given unitary evolution operator T may be expressed as $T = \exp(-i\tau H)$.

If S_1 is prepared such that $\rho_1 = P_\psi$, $\psi = a\alpha + b\beta$, and S_2 such that $\rho_2 = P_\alpha$, then at the onset of the measurement interaction $S_1 + S_2$ will be characterized by the global state vector $\psi\alpha = a\alpha\alpha + b\beta\alpha$.

Since

$$\begin{Bmatrix} T\alpha\alpha = \alpha\alpha, & T\alpha\beta = \beta\alpha \\ T\beta\alpha = \alpha\beta, & T\beta\beta = \beta\beta \end{Bmatrix} \tag{2}$$

the global state vector after the interaction will be $a\alpha\alpha + b\alpha\beta = \alpha\psi$. Thus the probability distributions for all S_2 observables after the interaction are identical to the distributions for corresponding S_1 observables before the interaction, a *probability match*; hence we have an operational measurement M_2. If one now asks for (i) the probability that $M_2(\sigma_z)$ will yield $+1$, the answer is certainly

$$\mathrm{Tr}[(P_{\alpha\psi})(1P_\alpha)] = \mathrm{Tr}[(P_{\psi\alpha})(P_\alpha 1)] = \mathrm{Tr}_1(P_\psi P_\alpha), = |\langle\alpha \mid \psi\rangle|^2 \tag{3}$$

where the first equality is a formal statement of the probability match on which the M_2 is based. On the other hand, if one asks which state S_1 is in after the M_2, the answer is that S_1 is certainly "in the state α," and this physical fact is completely independent of whether any individual run of the $M_2(\sigma_z)$ procedure yielded $+1$ or -1. Therefore, (ii) is not universally true, and it is not equivalent to (i).

Although a single measurement cannot determine an unknown quantum state, there is a procedure for doing so. It must be understood, however, that the role of *state* in quantum theory is played by the statistical operator ρ, which characterizes modes of *preparation*. Hence the problem of state determination must be posed as follows: given a preparation Π, how many data must be collected in order to find the statistical operator ρ which describes Π? One answer, developed by Band and me,[11] is that ρ may be determined from the mean values of a set of observables called the *quorum*. The size of a quorum depends on the dimensionality of the Hilbert space. In general, neither a single measurement result nor even an entire statistical collective of data from a single observable is sufficient information from which to deduce the quantum state. If the statistical operator turns out to be of the form $\rho = P_\psi$, then as a convention we may use this somewhat misleading but established jargon: Π prepares systems "in the state ψ."

Does Measurement Destroy Coherence?

This is not the place to enter again[9] into the interminable philosophical controversy that rages over the uniquely quantal phenomenon of interference of probabilities. There are a number of philosophers of science who may never accept the primitivity of the preparation-measurement format and who will therefore always see recondite paradoxes in the fabric of quantum physics.

This metaphysical debate has encroached upon theoretical physics itself to this extent: in terms of our simple two-dimensional example, it is often postulated that a measurement of σ_z upon S initially prepared in the pure state $\rho = P_\psi$, $\psi = a\alpha + b\beta$, *must* convert that state to the mixture $\rho_M = |a|^2 P_\alpha + |b|^2 P_\beta$. The rationale for this claim has much in common with the belief, discredited above, that each measurement act reveals which state vector S possesses. Thus, according to this view, the ensemble of measured systems should be a mixture, divisible into two pure subensembles characterized by the state vectors α, β uncovered by the measurements.

Since ρ incorporates phase relations between a and b not present in ρ_M, a measurement act which converts ρ to ρ_M is said to "destroy coherence."

It is indeed possible to envisage operational measurements M_2 which do destroy coherence. Among these is the famous two-slit gedankenexperiment with electrons, wherein the placement of counters to detect the electrons at the slits destroys the interference pattern that develops on the screen when the counters are absent.

This phenomenon is often given an extravagant subjectivistic interpretation according to which the interference is lost because the experimenter, in placing the counters, comes to know too much about the electrons; thus since quantum theory allegedly limits knowledge (complementarity) the experimenter must be penalized by forfeiture of his interference pattern. Fortunately there is, in terms of the preparation-measurement format, a more elegant and objective interpretation. Data can be altered in two ways: by changing the preparation or by measuring a different observable. In the two-slit experiment, with or without counters, the screen measures the same observable, *viz.*, electron position at some instant following preparation. Hence the reason that the screen exhibits different data in the presence and absence of counters at the slits must be that two distinct preparations are involved. Electron-source-plus-double-slit and electron-source-plus-double-slit-plus-counters are not identical preparations; therefore, measurements do not yield the same data for the two cases.

It is merely an *accidental* feature of the two-slit experiment that the difference in data with and without counters may be described as destruction of coherence due to measurement by the counters. Indeed, it is possible to conceive, as will be demonstrated below, of operational measurements M_2 which have the effect of *creating coherence*. Destruction of coherence is therefore not a universal characteristic of the measurement act.

To prove this, we consider first an M_2 which is mathematically similar to the two-slit experiment with counters but which involves only simple two-level systems. Let S_1 be a system prepared with $\rho_1 = P_\psi$, $\psi = a\alpha + b\beta$, a coherent superposition of σ_z-eigenstates. If S_1 interacts with another system S_2 (the "counters") in a manner described by the evolution matrix

$$(U) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \tag{4}$$

and S_2 is initially prepared with $\rho_2 = P_\alpha$, then since

$$\begin{Bmatrix} U\alpha\alpha = \alpha\alpha \\ U\beta\alpha = \beta\beta \end{Bmatrix} \tag{5}$$

the global state vector after the interaction will be $\Phi = a\alpha\alpha + b\beta\beta$. The reduced statistical operator for S_1 is then the partial trace over $\mathcal{H}_2$,

$$\rho_{1M} = \mathrm{Tr}_2 P_\Phi = |a|^2 P_\alpha + |b|^2 P_\beta \tag{6}$$

an incoherent superposition of σ_z-eigenstates. Thus the preparation of ρ_1 followed by the interaction with S_2 constitutes preparation of ρ_{1M}, which is quite distinct from ρ_1. Now, since there is a probability match between $M_1(\sigma_z)$ on S_2 after the interaction and $M_1(\sigma_z)$ on S_1 before the interaction, the procedure may be utilized as an $M_2(\sigma_z)$ for S_1; thus it is possible to say here that "measurement destroys coherence." But the coherence is destroyed not by the acquisition of knowledge but by the interaction, whether or not the interaction is actually applied as an operational measurement.

On the other hand, an act of measurement may occur which incidentally results in the *creation* of coherence. Suppose that S_1 is prepared initially in an incoherent state $\rho_1 = w_+ P_\alpha + w_- P_\beta$ and that S_2 is initially in a pure state $\rho_2 = P_\psi$, $\psi = a\alpha + b\beta$. The initial global state is then given by

$$\begin{aligned} \rho &= (w_+ P_\alpha + w_- P_\beta) P_\psi \\ &= w_+ P_{\alpha\psi} + w_- P_{\beta\psi} \end{aligned} \tag{7}$$

Let S_1 and S_2 interact according to evolution operator T, whose matrix was given by (1). Applying (2) to (7), we obtain for the postinteraction global statistical operator

$$\begin{aligned} \rho_M &= w_+ P_{\psi\alpha} + w_- P_{\psi\beta} \\ &= P_\psi (w_+ P_\alpha + w_- P_\beta) \end{aligned} \tag{8}$$

The state of S_1 has now been transformed *from* an incoherent superposition of σ_z-eigenstates *to* a coherent superposition of σ_z-eigenstates. Moreover, the interaction meets the test of establishing a probability match between preinteraction M_1s on S_1 and postinteraction M_1s on S_2, and may therefore be used as an M_2 for S_1. Of this measurement scheme it could truly be said that "measurement creates coherence"! The general proposition that measurement destroys coherence is therefore false.

Does Measurement Entail Uncontrollable Disturbance?

The normally cataclysmic impact of the measurement act on a microsystem is undeniable; however, characterization of the measurement disturbance as unpredictable, hence uncontrollable *in principle*, is somewhat ambiguous. Historically the uncontrollability doctrine arose in complementarist analysis of Heisenberg's microscope, wherein it was essentially noted that the final momenta in a single electron-photon collision cannot be predicted in advance. This is of course true, since classical mechanics is wrong and quantum mechanics predicts only probability distributions. Thus such discussions of uncontrollability really assert, in a colorful way, only this: quantum mechanics cannot in general make predictions concerning a single run of an experiment. In other words, quantum theory offers no classical predictions, hence no classical controllability of individual events.

However, this lack of control in the classical sense should not be construed to mean that the measurement act is quantally uncontrollable. On the contrary, it is possible to design operational measurements which do not disturb the quantum state at all or, to be precise, which leave the system after the measurement reprepared just as it was before the measurement interaction.

As an example, suppose observables of S_1 are to be measured by interaction with S_2. To devise the nondisturbing M_2, we first ascertain the quantum state of S_1 by measuring by any means available a quorum of observables. Assume for simplicity that the preparation of S_1 turns out to be pure, with state vector $\psi = a\alpha + b\beta$, a, b real. Let S_1 interact with S_2 according to the evolution matrix

$$(R) = \begin{pmatrix} a & b & 0 & 0 \\ 0 & 0 & a & b \\ b & -a & 0 & 0 \\ 0 & 0 & b & -a \end{pmatrix} \tag{9}$$

If S_2 is prepared with initial state vector α, the initial global state vector for $S_1 + S_2$ will be $(a\alpha + b\beta)\alpha = a\alpha\alpha + b\beta\alpha$. Applying (9), we find that

$$\begin{pmatrix} a & b & 0 & 0 \\ 0 & 0 & a & b \\ b & -a & 0 & 0 \\ 0 & 0 & b & -a \end{pmatrix} \begin{pmatrix} a \\ 0 \\ b \\ 0 \end{pmatrix} = \begin{pmatrix} a^2 \\ ab \\ ba \\ b^2 \end{pmatrix} \tag{10}$$

The postinteraction global state vector is therefore given by

$$a^2\alpha\alpha + ab\alpha\beta + ba\beta\alpha + b^2\beta\beta = (a\alpha + b\beta)(a\alpha + b\beta) \qquad (11)$$

The structure of (11) establishes that there is a probability match between the M_1s on S_2 after the interaction and corresponding M_1s on S_1 before the interaction and that the interaction also reprepares S_1 as it had been prepared originally. Hence we have a measurement act whose effect on the system has been "controlled" so well that the system emerges as though nothing had happened.

Finally, it is interesting to note that such a "nondisturbing" M_2 may be used to generate a time ensemble. Consider a large number N of systems initially prepared so that the initial global state vector is $(a\alpha + b\beta)\alpha\alpha \ldots \alpha[N \text{ factors}]$. If S_1 interacts successively with S_2, then S_3, etc., in the manner of evolution operator R, eventually the global state vector will be $(a\alpha + b\beta)(a\alpha + b\beta) \ldots (a\alpha + b\beta)$, provided that each S_n participates in no other interactions and that its own free evolution is stationary. There is now probability matching between the original M_1s of S_1 and the analogous M_1s of each S_n after the interaction. Suppose an $M_2(\sigma_z)$ is now performed simultaneously on $S_1, \ldots, S_N$; the yield will be a collective of σ_z-data which, according to the usual conventions, could be described theoretically as the results that would have been obtained by performing N $M_1(\sigma_z)$ acts simultaneously on S_1 after its original preparation.

Because latency, not possession, is the theoretical link between S_1 and σ_z, it is perfectly rational to imagine that simultaneous $M_1(\sigma_z)$ acts might yield different results. However, this does *not* refute, for example, the well-known experimental fact that an electron is never found in two places at once. The methods by which electron position is operationally measured involve interactions described by evolution operators more akin to T than R. Consequently, the Stern-Gerlach apparatus would never yield both $+1$ and -1 as simultaneous σ_z-data for a single system. Nevertheless, the formal possibility of a nondisturbing quantal measurement act calls attention in a new way to the many interesting but relatively unexplored questions regarding simultaneous measurements, sequential measurements, and macroscopic observations which continue to stimulate research into the foundations of physics.

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