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Superselection Rules in Quantum Theory: Part II. Subensemble Selection

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A dynamical analysis of standard procedures for subensemble selection is used to show that the state restriction violation proposal in Part I of the paper cannot be realized by employing familiar correlation schemes. However, it is shown that measurement of an observable not commuting with the superselection operator is possible, a violation of the observable restrictions. This is interpreted as supporting the position that each of these restrictions is sufficient but not necessary for the superselection rule. The results do constitute a proposal for superselection rule violation in theories requiring both restrictions, e.g., the axiomatic treatment by Bogolubov, Logunov, and Todorov. It is also concluded that superselection rules place restrictions on procedures for selective state preparations using correlations. More generally, it is conjectured that a mathematically conceivable decomposition of a given density operator does not necessarily represent a possibility for partitioning of the corresponding ensemble into subensembles by any physically realizable means.

1. INTRODUCTION

In Part I of this paper[1] it was shown that it is mathematically conceivable to decompose a given permissible density operator into a linear combination of not necessarily permissible density operators, where "permissible" means that $[\rho, Q] = 0$, Q being a superselection operator. By the usual interpretation of such a decomposition, an ensemble characterized by such a density operator can be considered as a mixture of subensembles whose density operators are those in the decomposition, the coefficients being interpreted as relative weights in the ensemble. We now ask if these decompositions can be physically realized, resulting in the preparation of illegal (nonpermissible) quantum states, which would be an instance of state restriction violation.[1]

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The problem is therefore the investigation of the possibility of selection of these nonpermissible subensembles. More significantly, we ask also if this state restriction violation can be accompanied by a concurrent observable restriction violation; only if this obtains will the superselection rule have been broken.[1] To our knowledge these questions have not been studied.

The selection scheme will involve a composite system $S + M$ consisting of subsystems S (quantum system of interest) and M (measurement apparatus). Presumably the initial global density operator ρ is permissible; Wick, Wightman, and Wigner (WWW)[2] have shown that the reduced density operator ρ_S for S is permissible if ρ is. They did not discuss the problem of selection of nonpermissible subensembles from an ensemble of S systems characterized by ρ_S. Assume that ρ embodies correlations between S observable A and an M observable B. Let Q denote an additively conserved superselection operator, such as charge,

$$Q = 1_S \otimes Q_M + Q_S \otimes 1_M \tag{1}$$

where 1_S and 1_M are the identity operators on the Hilbert spaces H_S and H_M for S and M; Q_S and Q_M are the corresponding charge operators. The permissibility of ρ implies that ρ commutes with Q, $[\rho, Q] = 0$. The extant formulations of superselection rules require that any observable defined on $H_S \otimes H_M$ commute with Q also; in fact, that is usually taken as an integral part of the definition of an observable. It will accordingly be assumed that $1_S \otimes B$ commutes with Q, thus $[B, Q_M] = 0$. At this stage, however, we will not require $[A, Q_S] = 0$, since it is logically possible that the correlation scheme, in selecting out illegal subensembles, in effect measures a *nonobservable*, an operator not commuting with charge, thus violating the superselection rule.

2. SPECTRAL DECOMPOSITIONS OF ρ_s

Consider the general form for the spectral expansion of the reduced density operator for S,

$$\rho_S = \sum_n w_n P(w_n) \tag{2}$$

Since Q_S commutes with ρ_S, it commutes with any function of ρ_S, in particular with any member $P(w_n)$ of the spectral family of ρ_S. Thus all the $P(w_n)$ are permissible and nothing unusual could be expected in a selection process. However, when ρ_S has degenerate eigenvalues, some of the $P(w_n)$ are multidimensional, thereby opening the door for a potentially more interesting sutuation: as shown in our work on alternative decompositions,[1] the spectral decomposition can then be written in a number of forms, including

$$\rho_S = \sum_{nd_n} w_n P_{\phi_{nd_n}} \tag{3}$$

where the $P_{\phi_{nd_n}}$ are the one-dimensional projectors onto the eigenvectors ϕ_{nd_n} spanning the nth eigenspace of ρ_S. The $P_{\phi_{nd_n}}$ are not not necessarily permissible, even though ρ_S is, and even though ϕ_{nd_n} is an eigenvector of ρ_S. A correlation scheme whose goal is the selection of a subensemble characterized by an illegal density operator $P_{\phi_{nd_n}}$ must involve an S non-observable A such that ϕ_{nd_n} and ϕ_{nd_n}' are A eigenvectors which belong to distinct A eigenvalues if $d_n \neq d_n'$. Obviously if $[A, Q_S] = 0$ this is not possible; the spectral family of A, each member of which commutes with Q_S, consists of only one-dimensional projections.

As it turns out, the explicit form of the alternative decomposition of ρ_S into nonpermissible projections will not have to be used: the prime concern is with the global state, its permissibility, and the demand that it embody correlations between A and B measurement results. As was mentioned earlier, $[\rho, Q] = 0$ and $[B, Q_M] = 0$ will be assumed, with allowance for $[A, Q_S] \neq 0$. It will also be recognized that not all the spectrum of A need be correlated with that of B: an eigenvector belonging to a correlated A eigenvalue a_k will be denoted by α_k, whereas an eigenvector belonging to a noncorrelated eigenvalue $\hat{a}_l$ will be denoted by $\hat{\alpha}_l$. The more familiar case occurs when ρ is a pure correlated state P_ψ with

$$\psi = \sum_n c_n \alpha_n \otimes \beta_n \tag{4}$$

where α_n and β_n are eigenvectors of A and B, respectively. ψ is of course a vector in the Hilbert space $H_S \otimes H_M$ for the composite system $S + M$. Since B commutes with Q_M, each B eigenvector β_n is an eigenvector of Q_M belonging to eigenvalue $q_M{}^n$ and hence

$$\begin{aligned} Q\psi &= \sum_n c_n(Q_S\alpha_n \otimes \beta_n + \alpha_n \otimes Q_M\beta_n) \\ &= \sum_n c_n(Q_S\alpha_n \otimes \beta_n + q_M{}^n\alpha_n \otimes \beta_n) \end{aligned} \tag{5}$$

If the Q superselection rule restrictions on quantum states are endorsed for the global state ψ, then $Q\psi = q\psi$ and thus

$$\begin{aligned} \langle \alpha_k \otimes \beta_l , Q\psi \rangle &= \sum_n c_n(\langle \alpha_k , Q_S\alpha_n \rangle + q_M{}^n \delta_{nk})\delta_{nl} \\ &= c_l(\langle \alpha_k , Q_S\alpha_l \rangle + q_M{}^l \delta_{lk}) \\ &= \Big\langle \alpha_k \otimes \beta_l , q \sum_n c_n\alpha_n \otimes \beta_n \Big\rangle = qc_k\delta_{kl} \end{aligned} \tag{6}$$

This equation leads to expressions involving the Q_S matrix elements in the "correlated part" of the A-representation,

$$c_l\langle\alpha_k, Q_S\alpha_l\rangle = 0, \qquad k \neq l \tag{7}$$

and

$$\langle\alpha_k, Q_S\alpha_k\rangle = (q - q_M{}^k) \equiv q_S{}^k \tag{8}$$

Let us closely investigate (7) under the assumption $\psi \neq 0$. Either $c_l = 0$ or $c_l \neq 0$. If $c_l = 0$, either $\langle\alpha_k, Q_S\alpha_l\rangle = 0$ or $\langle\alpha_k, Q_S\alpha_l\rangle \neq 0$ for any $k \neq l$; consider the second case. Then, interchanging k and l in (7) gives

$$c_k\langle\alpha_l, Q_S\alpha_k\rangle = 0, \qquad l \neq k \tag{9}$$

Under this assumption, the Hermiticity of Q_S implies

$$\langle\alpha_l, Q_S\alpha_k\rangle = \langle\alpha_k, Q_S\alpha_l\rangle^* = 0, \qquad k \neq l \tag{10}$$

Thus $c_k = 0$ for every $k \neq l$ and hence $\psi = 0$, a contradiction. Therefore (reductio ad absurdum) $\psi \neq 0$ and $c_l = 0$ imply $\langle\alpha_k, Q_S\alpha_l\rangle = 0$ for any $k \neq l$. Finally, consider the other possibility, $c_l \neq 0$; again, either $\langle\alpha_k, Q_S\alpha_l\rangle = 0$ or $\langle\alpha_k, Q_S\alpha_l\rangle \neq 0$ for any $k \neq l$. The second possibility contradicts (7): therefore (7) and $c_l \neq 0$ imply $\langle\alpha_k, Q_S\alpha_l\rangle = 0$ for any $k \neq l$. One infers from (7) that the nonvanishing of ψ entails that the Q_S matrix is diagonal in the correlated part of the A-representation.

If all of the spectrum of A is correlated, then the above result implies $[A, Q_S] = 0$ and that each A eigenvector is an eigenvector of Q_S. In that instance the correlation scheme selection procedure would yield subensembles characterized by the permissible projections P_{α_k}. However, suppose not all of the spectrum of A is correlated, so that

$$1_S = \sum_k |\alpha_k\rangle\langle\alpha_k| + \sum_l |\hat{\alpha}_l\rangle\langle\hat{\alpha}_l| \tag{11}$$

Then, since Q_S is diagonal in the correlated part of the A-representation,

$$Q_S|\alpha_k\rangle = \langle\alpha_k|Q_S|\alpha_k\rangle|\alpha_k\rangle + \sum_l \langle\hat{\alpha}_l|Q_S|\alpha_k\rangle|\hat{\alpha}_l\rangle \tag{12}$$

Therefore the A eigenvectors belonging to correlated A eigenvalues are legal only if Q_S has no nonvanishing matrix elements connecting the correlated and uncorrelated portions of the spectrum of A, i.e., $\langle\hat{\alpha}_l|Q_S|\alpha_k\rangle = 0$.

It turns out that the legality of the correlated A eigenvectors $|\alpha_k\rangle$ can be determined from the second equation for the Q_S matrix elements, (8).

Consider expanding $| \alpha_k\rangle$ in terms of the Q_S eigenvectors $| q_s d_{q_s}\rangle$, $Q_S | q_s d_{q_s}\rangle = q_s | q_s d_{q_s}\rangle$,

$$| \alpha_k\rangle = \sum_{q_s d_{q_s}} a_{kq_s d_{q_s}} | q_s d_{q_s}\rangle \tag{13}$$

Equation (8) becomes

$$\sum_{q_s d_{q_s}} q_s | a_{kq_s d_{q_s}} |^2 = q_S{}^k \tag{14}$$

Assuming normalization for the A eigenvectors,

$$\sum_{q_s d_{q_s}} | a_{kq_s d_{q_s}} |^2 = 1 \tag{15}$$

Combining (14) and (15) then gives

$$\sum_{q_s \neq q_S{}^k, d_{q_s}} | a_{kq_s d_{q_s}} |^2 (q_s - q_S{}^k) = 0 \tag{16}$$

This result should be independent of the particular labeling scheme for the Q_S eigenvalues: thus it should, for each k, hold for the scheme in which $q_S{}^k$ is the smallest Q_S eigenvalue, making $q_s - q_S{}^k \geqslant 0$. Therefore,

$$a_{kq_s d_{q_s}} = \delta_{q_s q_S{}^k} a_{kq_s d_{q_s}} \tag{17}$$

Comparing this with Eq. (13), it is seen that the A eigenvectors belonging to the correlated A eigenvalues are legal, i.e., they are Q_S eigenvectors. Therefore, even if not all the A spectrum is correlated with B measurement results, the subensemble selection procedure can still only yield permissible subensembles. It is seen now that Q_S has no nonvanishing A-representation matrix elements connecting the correlated and uncorrelated portions of the spectrum of A. That does not say, however, that A could not have nonzero off-diagonal matrix elements (A-representation) connecting uncorrelated A eigenvalues: thus $[A, Q_S] = 0$ does not necessarily follow. Therefore $[A, Q_S] = 0$ if all the spectrum of A is correlated. When the entire spectrum of A is not correlated, it is therefore possible that a nonobservable A is being measured, since A does not necessarily commute with Q_S. But this is clearly in a sense which the analysis shows does not cause difficulty with the state restrictions, i.e., measurement of a partially correlated A not commuting with Q_S by the correlation scheme does not lead to the selection of non-permissible subensembles. The correlated portion of the spectrum of any S observable always has legal eigenvectors; it follows that any illegal eigenvectors of such a nonobservable can never be correlated with the legal eigenvectors of a bona fide M observable B commuting with Q_M.

Consider now the general case of a mixed permissible global quantum state ρ embodying correlations between the spectra of the operators A and B used earlier. For such a density operator, the joint probability distribution $W(a_k, b_l; \rho)$ for simultaneous A and B measurements should be proportional to δ_{kl},

$$W(a_k, b_l; \rho) = \mathrm{Tr}(\rho P_{\alpha_k \otimes \beta_l}) \propto \delta_{kl} \tag{18}$$

Since ρ is permissible, it commutes with the total charge Q; thus ρ and Q share a complete orthonormal set $\{\psi_m\}$ of eigenvectors. Each ψ_m is legal since it is a Q eigenvector. They can be expanded in terms of the basis $\alpha_i \otimes \beta_j$ for $H_S \otimes H_M$ as

$$\psi_m = \sum_{ij} c_{mij} \alpha_i \otimes \beta_j \tag{19}$$

Moreover, the ψ_m can be used in a spectral expansion of

$$\rho = \sum_m w_m P_{\psi_m} \tag{20}$$

where each ρ eigenvalue w_m satisfies $w_m \geqslant 0$ since ρ is positive. Using these expressions, we obtain

$$W(a_k, b_l; \rho) = \sum_m w_m \,\mathrm{Tr}(P_{\psi_m} P_{\alpha_k \otimes \beta_l}) = \sum_m w_m \,|\, c_{mkl} \,|^2 \tag{21}$$

Since each factor in each term of this expression is $\geqslant 0$, it follows from the correlation requirement (18) that

$$c_{mkl} \propto \delta_{kl} \tag{22}$$

for every m, k, and l. Therefore each ψ_m exhibits the pure state correlated form (4). The analysis of the pure case may therefore be used as a lemma: thus once again the correlated A eigenvectors can only be legal, i.e., they must be eigenvectors of Q_S. All of the pure case conclusions then apply to the general case: so long as $[B, Q_M] = 0$, it is not possible to violate the state restrictions in a correlation scheme for selection of a nonpermissible S subensemble given any permissible global state ρ embodying A–B correlations; and it is possible to prescribe a measurement scheme for measurement of an S nonobservable without violating the state restrictions.

Thus, since the restrictions on observables are not as sacrosanct as heretofore thought, it is natural to continue the investigation of the spectral decompositions by asking if it is possible to violate the state restrictions under the assumption that there are no observable restrictions, i.e., relax the condition $[B, Q_M] = 0$. This would be a genuine superselection rule violation; we know of no previous proposal to violate both restrictions simultaneously.

This problem will be considered in the context of the correlation scheme; first consider the pure case with correlations,

$$| \psi \rangle = \sum_n c_n \mid \alpha_n \otimes \beta_n \rangle \tag{23}$$

where $| \psi \rangle$ is a permissible global pure state. Expand $| \alpha_n \rangle$ and $| \beta_n \rangle$ in terms of the eigenvectors of Q_S and Q_M, respectively

$$| \alpha_n \rangle = \sum_{qd_q} a_{nqd_q} \mid qd_q \rangle, \qquad | \beta_n \rangle = \sum_{pd_p} b_{npd_p} \mid pd_p \rangle \tag{24}$$

Therefore

$$\begin{aligned} | \psi \rangle &= \sum_n c_n \Big(\sum_{qd_q} a_{nqd_q} \mid qd_q \rangle \Big) \otimes \Big(\sum_{pd_p} b_{npd_p} \mid pd_p \rangle \Big) \\ &= \sum_{nqd_q pd_p} c_n a_{nqd_q} b_{npd_p} \mid qd_q \rangle \otimes \mid pd_p \rangle \end{aligned} \tag{25}$$

The permissibility of $| \psi \rangle$ means that it is an eigenvector of total charge, $Q \mid \psi \rangle = Z \mid \psi \rangle$. Therefore

$$\sum_{nqd_q pd_p} c_n a_{nqd_q} b_{npd_p} (q + p) \mid qd_q \rangle \otimes \mid pd_p \rangle = Z \mid \psi \rangle \tag{26}$$

This can only be satisfied if $(q + p) = Z$ independent of n. The permissibility of $| \psi \rangle$ therefore entails that $| \psi \rangle$ have the form

$$| \psi \rangle = \sum_{\substack{nqd_q pd_p \\ (p+q)=Z}} c_n a_{nqd_q} b_{npd_p} \mid qd_q \rangle \otimes \mid pd_p \rangle \tag{27}$$

This Q eigenvector can be in the correlated form given by (25) only if

$$a_{nqd_q} b_{npd_p} = \delta_{p+q,Z} a_{nqd_q} b_{npd_p} \tag{28}$$

for every n, p, and q. Let $a_{nq'd_{q'}}$ and $b_{np'd_{p'}}$ denote any two nonvanishing coefficients, for some n, i.e., $q' + p' = Z$. Consider any $q'' \neq q'$; then $q'' + p' \neq Z$ and thus

$$a_{nq''d_{q''}} b_{np'd_{p'}} = 0 \tag{29}$$

Since $b_{np'd_{p'}} \neq 0$, it follows that $a_{nq''d_{q''}} = 0$ for any $q'' \neq q'$. Hence a_{nqd_q} can be nonvanishing for at most one value of q, say $q = q'$,

$$a_{nqd_q} = \delta_{qq'} a_{nq'd_{q'}} \tag{30}$$

for any n and q. A similar argument leads to the result, for any n and p,

$$b_{npd_p} = \delta_{pp'} b_{np'd_{p'}} \tag{31}$$

Therefore all the correlated eigenvectors $|\alpha_n\rangle$ and $|\beta_n\rangle$ are legal, and hence an illegal S subensemble could never be selected. Nor is it possible to measure an M observable B whose correlated eigenvectors are illegal; note that this does not require $[B, Q_M] = 0$.

The main result of the above analysis is that the permissible global pure state embodying correlations does not admit selection of illegal S subensembles. A result also emerged which permits the extension of this pure case result to the general global mixture (permissible, correlated): any Q eigenvector in the correlated form (23) involves only legal A and B eigenvectors $|\alpha_n\rangle$, $|\beta_n\rangle$, independent of whether or not either A or B commutes with Q_S and Q_M, respectively. The extension to the general case draws on the earlier result [cf. following (18)] that if ρ is permissible and embodies A–B correlations, then ρ and Q share a set of legal eigenvectors exhibiting the pure state correlated form (23). This result was also independent of whether or not A and B commuted with Q_S and Q_M, respectively. Therefore the most general case involving correlations can never yield an illegal quantum state, provided the initial global state is legal: there is no conceivable correlation scheme for selecting an illegal S subensemble, even though the permissible reduced density operator ρ_S for the ensemble of S subsystems can be spectrally decomposed into a mixture of not necessarily legal density operators. The superselection rule apparently places restrictions on the possible selection procedures; thus, not every mathematically conceivable decomposition of a given density operator represents a physically possible partitioning of the corresponding ensemble into subensembles.

3. NONSPECTRAL DECOMPOSITIONS OF ρ_S

The above demonstrations are limited to the situation wherein the correlated A eigenvectors α_k are mutually orthogonal. The reduced density operator for S in these circumstances was in the form of a spectral expansion

$$\rho_S = \sum_k w_k P_{\alpha_k} \tag{32}$$

Thus the spectral family of ρ_S consisted of some (note necessarily all) members of the spectral family of A. The earlier work on alternative *spectral* decompositions[1] when ρ_S had a degenerate spectrum indicated a possibility that α_k could be illegal; thus the selection of nonpermissible subensembles was mathematically indicated. That work also indicated that *nonspectral* expansions of a permissible ρ_S could take the form in (32) even when ρ_S has a nondegenerate spectrum; in the nonspectral case, however, the α_k are no longer orthogonal and the above demonstrations would not suffice. Thus let us now consider the selection problem anew, relaxing the orthogonality requirement so as to include the broad area of the nonspectral expansions.

It is now asked if the correlation scheme employed above can be used to select the nonpermissible subensembles indicated in the nonspectral decompositions. This time two S observables will be used, denoted A and C: A has eigenvectors α_k appearing in a nonspectral expansion of ρ_S in the form given in (32), but where the α_k are no longer eigenvectors of ρ_S and are not necessarily orthogonal; C has a complete orthonormal set of eigenvectors γ_k. Nothing need be said regarding legality of the α_k or γ_k for what follows. As before, a single M observable B with legal eigenvectors β_K will be used.

Consider the pure case embodying correlations; assume the general form of such a state is

$$\psi = \sum_n c_n \alpha_n \otimes \beta_n \tag{33}$$

Since the β_n are orthogonal, the reduced density operator is given by

$$\rho_S = \mathrm{Tr}_M(P_\psi) = \sum_n | c_n |^2 P_{\alpha_n} \tag{34}$$

which is a nonspectral expansion of ρ_S if some of the α_n are not ρ_S eigenvectors. If the α_n are mutually orthogonal, the above decomposition is spectral. It will now be shown that the presence of correlations demands mutual orthogonality of the α_n.

The joint probability $W(a_k, b_l; \psi)$ that simultaneous A and B measurements yield the results a_k, b_l for an ensemble of composites $S + M$ prepared in the manner characterized by ψ is given by

$$\begin{aligned}
W(a_k, b_l; \psi) &= \mathrm{Tr}(P_{\alpha_k \otimes \beta_l} P_\psi) \\
&= \sum_{mn} c_m c_n^* \, \mathrm{Tr}(| \alpha_k \otimes \beta_l\rangle\langle\alpha_k \otimes \beta_l | \alpha_m \otimes \beta_m\rangle\langle\alpha_n \otimes \beta_n |) \\
&= \sum_{mn} c_m c_n^* \langle\alpha_k | \alpha_m\rangle \, \delta_{lm} \\
&\quad \times \sum_{pq} \langle\gamma_p \otimes \beta_q | \alpha_k \otimes \beta_l\rangle\langle\alpha_n \otimes \beta_n | \gamma_p \otimes \beta_q\rangle \\
&= \sum_{mn} c_m c_n^* \langle\alpha_k | \alpha_m\rangle \, \delta_{lm} \delta_{nl} \langle\alpha_n | \sum_p P_{\gamma_p} | \alpha_k\rangle \\
&= | c_l |^2 \, |\langle\alpha_k | \alpha_l\rangle|^2
\end{aligned} \tag{35}$$

For correlations to exist between A and B measurement results this joint distribution should be proportional to δ_{kl}. Inspection of the above expression shows that the required correlations can only occur if the α_k are mutually orthogonal. [Another conclusion might be that (33) is *not* the general correlated pure state for the case of nonorthogonal α_n and β_n. We have not investigated the possibility of alternative pure state forms in this situation.]

This argument is easily extended to the general case of a mixed permissible global state ρ embodying A–B correlations. The analysis leading to (22) applies in this case: each member of the eigenvector set which ρ and Q have in common as a result of their commutativity exhibits the pure state correlated form (33). Thus the A eigenvectors belonging to the correlated part of the spectrum of A are mutually orthogonal by the argument leading to (35).

Therefore the nonspectral decomposition cannot be exploited to violate the state restrictions or the superselection rule itself in a correlation scheme for subensemble selection.

4. SUMMARY AND CONCLUSIONS

The physical problem of subdivision of an ensemble with permissible quantum state ρ_S into nonpermissible S subensembles was considered after showing[1] the mathematical consistency of such a situation. To do this, ρ_S was considered to be the reduced density operator for the ensemble of S subsystems of an ensemble of composite systems $S + M$ with a global quantum state ρ which was assumed to be permissible and to embody correlations between S and M observables A and B.

WWW showed that ρ_S is always permissible if initially only permissible states are given. Now added to this is the result that any actual S subensemble is always permissible, even though ρ_S can in general be decomposed into nonpermissible density operators, i.e., even though the S ensemble characterized by ρ_S can be mathematically conceived of as a mixture of nonpermissible S subensembles. Thus the proposal for state restriction violation advanced in Part I of this paper[1] is untenable if the most general method for subensemble selection involves use of correlations.

It was determined that it is not possible to select an illegal S subensemble even if allowances for measurement of nonobservables are made; for, if the global state is permissible and embodies correlations, the eigenvectors belonging to correlated eigenvalues of S and M observables are always charge eigenstates. Thus, even though measurement of observables not commuting with charge are possible, they can never lead to violation of the state restrictions if the initial global state is permissible. Since both the state restrictions and the observable restrictions are individually sufficient to ensure validity of the superselection rule postulate,[1] it follows that the correlation scheme herein studied can never be used to violate the superselection rule, given an initial permissible global state.

The logical status of the state and observable restrictions clearly plays a central role in deciding whether or not a superselection rule has been violated. It was pointed out in Part I of this paper that the logical status of these restrictions differs among the two schools of thought. In the Bogolubov, Logunov, and Todorov (BLT)[3] formulation the conjunction of these restrictions is necessary and sufficient for the superselection rule; in the WWW formulation as interpreted herein, this is replaced by a disjunction. To violate a WWW superselection rule both restrictions must be violated; the task in the BLT context is clearly easier. In this part of the paper it was shown that a nonobservable can be measured using the correlation scheme, but *not* with concurrent selection of an illegal subensemble. Thus only the BLT superselection rule was violated. Since the prescription for nonobservable measurement in the presence of state restrictions is consistent with non-violation of the WWW superselection rule as interpreted here, it is concluded that the BLT axiomatics is too strong.

Since it is wrong to contend that both the observable and the state restrictions are necessary conditions for a superselection rule, one is free to consider all Hermitean operators as possible observables provided any quantum state is suitably restricted, i.e., $[\rho, Q] = 0$, and vice versa. The usual "proof" that the state restrictions imply observable restrictions is invalid since it confuses the measurement and preparation acts by employing the projection postulate, as first pointed out by Park.[4]

Since superselection rules do indeed place restrictions on procedures for selective state preparation using correlations, we are led to conjecture more generally that in their presence a mathematically conceivable decomposition of a given density operator does not necessarily represent a possibility for partitioning of the corresponding ensemble into subensembles by any physically realizable means. Strictures on subensemble selection procedures have been considered elsewhere in a thermodynamic context by Hatsopoulos and Gyftopoulos.[5]

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