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Another Look at Complete Positivity in Generalized Quantum Dynamics: Reply to Raggio and Primas

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In this rejoinder to a critique by Raggio and Primas of our paper, "On Completely Positive Maps in Generalized Quantum Dynamics," we acknowledge that, contrary to our original assertion, the Bloch equations are indeed completely positive. We then explain briefly why this modification of our analysis does not alter its main conclusions.

As a byproduct of a continuing search for possible generalizations of quantum dynamics that might accommodate in particular the entropy-increasing motions mandated by thermodynamics but forbidden by unitary mechanics, the present authors published last year an analysis[(1)] of completely positive maps. That paper has generated correspondence both praising and condemning its content. Unfortunately our remarks apparently struck some readers as a somewhat inflammatory deprecation of phenomenological studies based on the theory of such maps, even though we explicitly noted that the elegant work[(2)] of Gorini *et al.* "delineates an important family of motions which are correctly describable by completely positive maps." In fact our intent was never to comment adversely or otherwise on the methodology of such fields as nuclear magnetic resonance, but rather to assess the theoretical significance of complete positivity as a possible basis for a generalized quantum thermodynamics.

One section of our paper dealt with the interpretation of a theorem of Gorini *et al.* that provides necessary and sufficient conditions for completely

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positive maps when the system of interest has a two-dimensional Hilbert space. We proved that these conditions yielded an equation of motion differing from the Bloch equations and then deduced that the latter must not be completely positive. In their recent critique of our article, Raggio and Primas[3] objected to this statement and demonstrated that the mathematical form of the Bloch equations does in fact satisfy the definition of complete positivity; it is then only natural for them to conclude that our entire analysis consequently collapses. The situation is actually a bit more subtle.

First let us acknowledge that Raggio and Primas are correct on the technical point that the Bloch equations are completely positive. In stating the opposite we misinterpreted our own mathematical analysis. In the context of our investigation the problem had been this: Is the concept of complete positivity sufficiently powerful to determine the new laws of motion for general quantum thermodynamics? We were accordingly quite excited by the prospect seemingly offered by the aforementioned theorem of Gorini *et al.*, which we had thought of as a derivation of the Bloch equations from the principle of complete positivity. Further consideration revealed problems with this view, fully discussed in the Appendix of our paper, which contains the mathematics we misinterpreted as establishing that Bloch's equations were not completely positive. The conclusion we should have drawn from this analysis was simply that the principle of complete positivity is not sufficient to determine Bloch's equations. Therefore our claim that comparison of the work of Gorini *et al.* with Bloch's theory does not support"... the hypothesis that completely positive dynamical maps might be useful in describing the quantal motion of subsystems..." remains valid in the context in which we stated it.

If the conditions required by complete positivity were necessary and sufficient to derive the Bloch equations, then complete positivity would offer an excellent foundation for a new, non-Hamiltonian dynamical postulate of the type that we and others[4] are seeking. Unfortunately, the Bloch equations emerge from complete positivity only when a special additional condition ($\mathbf{m}_0 \times \mathbf{h} = 0$) is imposed. In fact it is known that complete positivity also includes ordinary Hamiltonian motion as a special case. Thus, for us, complete positivity is essentially a neutral criterion, favoring neither entropy-conserving nor entropy-increasing forms of mechanics.

Though we regret the misstatement about the Bloch equations and are pleased to clarify the matter, nevertheless our final conclusion is in essence no different: Despite its formal beauty and apparent phenomenological utility, the completely positive map is not an adequate construct for the formulation of fundamental dynamical principles.

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