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Preparation and Measurement in Quantum Physics

James L. Park[1] and William Band[1]

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To honor Henry Margenau on the occasion of his 90th birthday, we attempt in this essay to integrate certain aspects of the physics, philosophy, and pedagogy of quantum mechanics in a manner very much inspired by Margenau's idealist scientific epistemology. Over half a century ago, Margenau was perhaps the first philosopher of science to recognize and elaborate upon the essential distinction between the preparation *of a quantum state and the* measurement *of an observable associated with a system in that state; yet in contemporary quantum texts that distinction rarely receives adequate emphasis even though, as we demonstrate, it may be explicated through a series of simple illustrations.*

1. INTRODUCTION

To honor Henry Margenau on the occasion of his 90th birthday, we attempt in this essay to integrate certain aspects of the physics, philosophy, and pedagogy of quantum mechanics in a manner very much inspired by Margenau's original contributions[1,2] to the idealist scientific epistemology of the quantum era. Stated briefly, his underlying philosophical system owes much to Kantian idealism, but no use is made either of absolute *a priori* propositions or of any *ding an sich* beyond human experience. Thus, in particular, quantum physics is not taken to be descriptive of any independent, external world of the sort commonly posited by Occidental materialist-reductionist scientists and philosophers. Instead the noumenal realm of physical reality is populated with empirically verified and metaphysically certified constructs epistemically linked by rules of correspondence to the phenomenal world. The class of such constructs is quite rich, encompassing not only ordinary objects like rocks and flowers but also scientific abstractions like black holes, quark fields, and state vectors.

[1] Department of Physics, Washington State University, Pullman, Washington 99164.

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Back in the formative years of the new quantum theory, Margenau was perhaps the first philosopher of science to recognize and elaborate upon the essential distinction between the *preparation* of a quantum state and the *measurement* of an observable associated with a system in that state; but, fundamental as it is, that distinction has yet to penetrate appreciably into the domain of elementary or graduate texts on quantum theory. It is not unusual to find excellent didactical presentations of the quantal algorithm embedded in a kind of structureless philosophical void which offers the student no clue as to the empirical significance of the formalism. And even when the situation is not quite that bad, students regularly confront, in lieu of an epistemologically sound aproach, a pseudohistorical rerun of some antique gedankenexperiments—which inspired Heisenberg but have confused his successors—plus some glib and fantastic talk about duality and matter waves, two notions utterly foreign to the modern probabilistic quantum theory. The quantum physical electron, for example, should not be dually and absurdly construed as being a particle *and* a wave, but as neither a particle *nor* a wave. Matter waves bear about the same relation to contemporary quantum theory as do undulations of the luminiferous ether to modern electrodynamics. Consequently, it is incumbent to an unusual extent upon the teacher of quantum mechanics to lecture well beyond the formalistic boundaries of even the best texts if the physical meaning of the theory is to be conveyed. Otherwise students may simply memorize incomprehensible assertions and propagate them as dogma to future generations, a process already too evident in the annals of quantum mechanics. We believe that the student of quantum theory can successfully weather the above-mentioned and other common affronts to his intellect if at some point in the educational process, preferably early, he is introduced to the *preparation-measurement* format of experimental science and then immediately taught the particular manner in which quantum theory copes with physical problems by employing that framework.

2. THE PREPARATION-MEASUREMENT FORMAT

In any scientific theory which employs the concept of probability in such a manner that its numerical values are regarded as objectively verifiable predictions, the operational meaning of probability is given by the familiar relative frequency definition: to say that W is the probability that an event E will occur means empirically that for a sufficiently large number N of identical trials, the event E will occur WN times. Now it is obvious that attribution of a specific value to W can be a testable assertion only if arrangements or circumstances are agreed upon *a priori* which establish unequivocally what is to be meant by identical trials. There must be instructions describing repeatable acts or operations which set the stage for the occurrence or nonoccurrence of E. The number W is then characteristic of the overall *preparation* which can be repeated to generate a *statistical ensemble* of N experiments in each of which E is searched for and in WN of which E is found.

From the advantageous perspective afforded by hindsight, the foregoing truisms about probability and statistics may seem so self-evident as to be unworthy of special emphasis. However, in the turbulent developmental years for modern quantum physics, it was precisely the failure to enunciate forcefully the crucial significance of this concept of preparation that resulted in the philosophical befuddlement whose legacy continues to permeate the textbooks.

Ironically, von Neumann's early treatise on quantum foundations encompassed both an ensemble viewpoint[(3)] in which the preparation concept was implicit as a natural concomitant of statistics and an acausal measurement intervention process[(4)] based upon the projection or wave packet reduction postulate in which probabilities mysteriously belong to single elements of the ensemble. Unfortunately, the latter notion became an institution, while the former rational analysis was ignored. In the aftermath of the Einstein[(5)]–Bohr[(6)] controversies, Margenau[(7)] became the first to point out the absurdity of the projection postulate and to emphasize the significance of the *preparation* concept as the key to disentangling quantum physics from the philosophical morass into which its foundations had been thrust.

Nevertheless, to this day the preparation process[(8)] is still often misnamed "measurement," even though that term is sorely needed elsewhere in its traditional role, where it is distinguished from all other operations upon physical systems by one universal trait: *the measurement act yields a numerical datum* linked theoretically to the measured observable. The emergence of a datum is the typical event E with which physical theory, including quantum theory, deals. The repeatable *preparation* which generates the statistical ensemble from which the data are collected certainly need not, and in general will not, be an act of *measurement*. Yet this is the impression given by many texts, where overdrawn analyses of the Stern–Gerlach experiment, for example, attempt to persuade the reader that measurement is filtration and that filtration is preparation.

Such unjustified idealizations long ago germinated the well-known paradoxes concerning the alleged collapse of the wave function, and voluminous quasimystical discussions of the role of consciousness in determining the quantum state. In this paper we shall bypass these paradoxical issues, and describe instead some simple examples of the preparation of quantum states, and subsequent measurement procedures, in order to show that quantum theory, when properly interpreted, is manifestly consistent with the common sense notions usually taken for granted by experimental physicists.

On the formal side, in lieu of any obscurantism about duality or matter waves, we prefer the following three introductory axioms that specifically mention, among other primitive physical terms like *system* and *ensemble*, the distinct ideas of *measurement* and *preparation*:

I. For each physical *system* there is a Hilbert *space*.

II. Each *Hermitian operator* A on that space represents an *observable*; i.e., each such operator is associated with a class of *measurement* procedures.

III. For each *preparation* sheme there is a *statistical operator* ρ such that the arithmetic mean of A-data gathered from an *ensemble* generated in the manner represented by ρ is given by $\mathrm{Tr}(\rho A)$.

The first two axioms are not unusual, but Axiom III replaces a more common proposition that attempts to regard quantum states as properties possessed by individual systems. Axiom III is admittedly quite abstract, but at least it is distinctively *physical* in that it refers to the preparatory stage of an experiment cast in the preparation-measurement format described above. By contrast, we would maintain that to attribute, as textbook jargon so often does, a ket vector or a wave function to each individual system rather than to its preparation is quite *unphysical* and, in the context of a statistical theory, even irrational. When a preparation is characterized by a projector $\rho = |\psi\rangle\langle\psi|$, the state vector ψ, like ρ, refers to the preparation; to say of an ensemble characterized by ψ that each system "is in state ψ" is a spurious extrapolation incompatible with the profoundly statistical context in which the construct ψ arises.[9]

3. PEDAGOGICAL ILLUSTRATIONS

The development of an intrinsically quantum mechanical intuition unencumbered by the weight of outmoded classical misconceptions—an intuition grounded in the idealist epistemology long advocated so fervently by Henry Margenau—is hampered by the linguistic fact that normal descriptive prose inherently reflects, or at least suggests, the traditional materialist-reductionist world view. Nevertheless, we believe that an idealist quantal intuition can be acquired and that it can even be expressed in common-sense language through critical consideration of a graduated series of pedagogical illustrations of the preparation-measurement format. To that end, we conclude this essay by offering such a series.

A. We have an English oak tree in the garden, and we are interested in oak leaves. The observables might be size, shape, and color of the oak leaf. The tree itself prepares an ensemble of oak leaves which we are free to examine. The preparation proceeds with time and season. We may select the preparation that is completed by the date March 21st, or we may wait until September 22nd. In either case, the preparation precedes the measurement we may wish to make on the ensemble. Such measurements involve statistical problems. Ideally we should make them on every member of the ensemble and subject the results to statistical analysis. In practice there is too much foliage on the tree and we resort to selective procedures. Perhaps we are interested only in mature leaves, as of March 21st; thus we shall select a smaller ensemble, or a subensemble. And we regard this as a selective phase of the preparation. We might elect to define mature leaves as those having an area greater than a certain size, in which case the selection of the ensemble would proceed along with some of the measurements. Nevertheless the complete preparation must be at hand before we can subject the measurement results to statistical analysis.

As an alternative selective preparation we may arbitrarily take one branch of the tree and make measurements on every leaf of that branch. The entire tree is now regarded as a "mixture" of all the branch-ensembles. One can imagine many other sampling techniques. The point to be emphasized is that whether the preparation provided by nature is accepted, or whether we further make selective or filtering operations before accepting the ensemble, the complete prearation must be prescribed and at hand before the results of subsequent (or concurrent) measurements can be interpreted. And the results of the measurements are characteristic of the complete preparation procedure.

B. Physics, being a laboratory science, is more apt to present us with contrived preparation schemes. However, we shall begin with an example where nature does provide physicists with a prepared ensemble, which, as with the oak tree, can then be further refined by selective devices.

We are interested in cosmic protons observable from a space laboratory outside of Earth's atmosphere. In the laboratory we have devices that can record the direction and magnitude of the linear momentum of the observed protons. We may first define our ensemble as all cosmic protons present in the solar system at a certain time period, but to observe such an ensemble would require a large fleet of space ships scattered about the solar system. It would be more practical to restrict the ensemble to protons observed within a spherical shell around Earth, within which one space lab can orbit. The total ensemble in the solar system would be a mixture of such subensembles in different parts of space.

A further selection may be imposed: we may be interested only in those protons that have momentum within some small solid angle in a fixed direction relative to the solar system; and finally we may from this ensemble again select only those with a given magnitude of momentum within a given small range of values. We call this an almost "pure" state preparation because the result of any subsequent momentum measurement has an almost foregone conclusion. The more narrowly one prescribes the momentum by the preparation, the more precisely one can predict the result of a measurement. The pure state (ensemble) is an ideal limit where the result of a measurement can be foretold exactly.

When we have completed this selective preparation of a pure state, we have, according to conventional wisdom, already performed the measurement. In fact, this is where the conventional treatments go astray; for we have *not* quite completed the measurement. The actual arrival of the particle must next be detected by the counter. Only then may we assert that this particle was indeed a member of the selected ensemble. Note the past tense. After counting, the article has been subjected to a violent interaction, and it is removed from the ensemble by the observation. If the ensemble were a very small collection, such a process would be awkward for any statistical analysis. Our ensembles are so populous, however, that removal of a few individuals does not affect the subsequent probabilities.

Conventional discussions based on the von Neumann mathematical scheme stopped at the pure state preparation and asserted that the measurement act had left the particle in the pure state corresponding to the result of the measurement. This is really nothing worse than a semantic error. It is much better to use the word *preparation* here instead of *measurement*. In this way we can include discussions of preparations of mixed and of superposition states in a rational fashion, without doing violence to common-sense notions about measurement.

C. Let us return to our oak tree, and examine the ensemble on September 22. From our casual observation over past autumns we expect that many of the leaves will have turned red, and that most of these will be in part red and in part green. Now imagine that we are color blind, and that we are provided with a device that can read out either "red" or "green" but not both, and that if presented with a pied object, only chance will decide its output. The resulting ensemble of measurement results will seem to indicate a mixture of red leaves and green leaves, but in fact the ensemble cannot be divided into such parts, one of all green, and the other of all red leaves. The mixing occurs within the individual systems although our measurements with this particular device fail to show that.

Now let us use a device that reads out the ratio of red area to green area for each leaf. We may then discover a distribution of area ratios within the ensemble, and this could be regarded as a mixture of ensembles each having a specific area ratio. Any subensemble of a given area ratio would be a pure state relative to this device, but if examined by our previous red-or-green detector, it would still look like a mixed ensemble, which however it is not. It is the classical analog of what is known in quantum mechanics as a superposition state.

This example is about as near as we can get to a classical analog of the quantum idea of superposition, and it of course has its limitations. But it does point up the fact that it is a limitation of the measurement device that makes the superposition state indistinguishable from a mixture. If we ask the right questions—use the area ratio detector on the tree—we get a pure state response.

It is also important to realize that in this—and in other classical examples—we generally have a great deal of background information from previous measurements; and it is often not realized that if we were presented with an ensemble about which we knew absolutely nothing—that is, we have never performed measurements before on similar ensembles—the nature of the preparation has to be inferred from subsequent measurements. We do not know whether the ensemble is pure or mixed, and we cannot tell simply by measuring a few obvious observables. If in the above example we had not discovered, or invented, the area-ratio detector, we would not know that the ensemble is a pure state. The number and nature of observables required to determine completely the nature of any given preparation may be exceedingly large and complicated. A systematic discussion of this problem, or at least a first effort in this direction, has been given elsewhere.[10]

D. Preparation of momentum eigenstates.

(i) Preparation of a neutron in a state of definite momentum. The ensemble of neutrons prepared in and emerging from a pile may be examined directly, and a wide distribution of momenta is observed. The ensemble is a mixture. Let the preparation now be extended to include a collimator between two appropriately synchronized choppers in an effort to select a small range of momenta from the original ensemble.

Practical limits obviously exist to the purity of the momentum thus secured, apart from the fact that the finite time between cut-offs inevitably prohibits monochromatic purity of the de Broglie wave representing the neutron. But at least in principle one can obtain a pure monochromatic wave, hence generate a pure momentum state ensemble of neutrons. Our confidence in the efficacy of this experimental procedure is at best indirect; it is based on previous experience where subsequent measurements have already been made to test the results. In an experiment requiring neutrons of specified momentum, this preparation is carried out in order to have available a time ensemble of neutrons with the definite momentum which we know without having to measure it. Indeed to measure it would ruin the experiment of interest.

(ii) Preparation of an electron in a state of definite momentum. Boiling electrons off a hot filament constitutes a preparation providing a mixed ensemble with a nearly Maxwellian distribution of momentum eigenstates. To further purify the ensemble, let the electrons enter an accelerating electric field, and then a transverse magnetic field. The desired momentum is then selected by filtering out all those undesired momenta having the wrong curvature in the magnetic field. Again there are obvious limits to the precision, but conceivably it would be possible to get a pure momentum state. Again we do not make any measurement in the course of the preparation. Our confidence in the method is based on previous measurements following similar preparations.

E. Preparation of a single crystal in its ground state. The crystal is placed in a vacuum enclosed in a perfectly absorbing wall that is maintained as close as possible at 0 K. Spontaneous emission of photons will eventually reduce the crystal to its ground state. Again there are limits to this precision. If the walls are not exactly at 0 K the electromagnetic field in the vacuum will contain a statistical distribution of photons which can occasionally excite the crystal. If the walls are not perfectly absorbing, the photons emitted by the crystal will in part return to excite the field and so maintain the crystal in an excited state. But it is conceivable in a limiting sense to secure such a crystal in its ground state.

There is a glaring difference between this example and the previous ones. There we had a large number of objects, protons, leaves, etc., while here we have only a single crystal. If we make a measurement on the crystal (cf. example F below) we must reprepare the same crystal before making another measurement, and repeat this many times in order to build up a statistically significant collection of data from which to compute averages and distributions. It is this ensemble of results of measurements on a repeatedly prepared crystal that characterizes the preparation procedure, and it is to this ensemble that the quantum state ρ of the crystal refers.

Incidentally, the crystal is a very large agglomeration of atomic parts whose ground state is perhaps unique; but any very slight excitation already involves many millions of different possible atomic excitations that cannot be distinguished from each other by the preparation procedures we have so far visualized. Thus nearly pure ensembles of the crystal are actually mixtures of very large numbers of subensembles all of which appear macroscopically the same. The detailed structure of the ensemble is unknown even though the preparation procedure is well defined experimentally on a macroscopic scale. The connection between this and the information theory interpretation of entropy is obvious, but will not be pursued further in this paper.

F. Measurement of crystal lattice structure at 0 K. Imagine the walls of our evacuated enclosure to have a finely manufactured structure such that each absorbing spot is connected outside with a counter. After waiting long enough to cool down the crystal, we open a small hole and admit a single neutron prepared as in example D. Every counter reports either "yes" or "no." This measurement is repeated; every time we wait long enough to be sure the crystal has returned to its lowest state, and reintroduce the neutron (another neutron) similarly prepared. A sufficient number of such measurements, each time preceded by the proper preparation procedure, builds up the diffraction pattern from which the crystal structure can be derived.

Again there are obvious limits to precision in this measurement procedure. But the logic of the method should be clear. We first prepare, in initial states of interest, all systems to be employed, then let them interact, and finally perform desired measurements. This prototype serves also to emphasize that every physical measurement can be cast in the form of yes-or-no questions.

Preparations are entirely different: they are based on theoretical analysis, backed up by previous experience that may involve measurements that we do not need, or indeed must not make, if we are to perform successfully the measurements we are currently interested in making.

G. Polarization of photons. We use an old-fashioned ideal polarizer that permits one to see both the transmitted and the reflected photons, without absorption. We prepare a beam of incident photons from some low-intensity source which we believe has characteristics that do not change with time. (This in itself involves a background of experience with the type of experiment we are about to describe.) We set up counters C_t and C_r to receive and record the arrival, after interacting with the polarizer P, of transmitted and reflected photons, respectively. The ratio of the numbers of counts n_t and n_r over a long period of time is noted. The ratio is believed to be characteristic of the source—again based on previous experience. If upon placing a second identical polarizer P¢ between P and C_t, no change in the long term ratio of n_t to n_r is observed, we are encouraged to believe that the first P has produced a purely polarized beam of photons. Note that the counting of n_t and n_r represents a measurement; and this was necessary to convince us that the device actually produces a polarized beam. When this conviction has been established, we remove the counters, and use the polarizer alone to prepare what we now believe to be a pure polarized beam—this is the preparation that must precede whatever other experiments and measurements we may wish to do on the prepared polarized beam.

H. Measurement of optical birefringence of a crystal at 0 K. The crystal is prepared as in example E. The beam of photons is prepared as in example G. The beam passes through the crystal and then through another polarizer followed by a counter, or set of counters strategically placed. The mutual orientation between crystal and polarizers modifies the counts received; i.e., a correlation is obtained between mutual orientation and counter readings, from which the refractive properties of the crystal can be derived. Every measurement is a yes-or-no datum on each counter. Any "yes" means that the photon has been recorded, and in the process destroyed. A new photon is prepared for every such bit of information.

I. Photon polarization, continued. Use a polarizer P_1 to prepare a polarized beam, and pass it through a polarizer P_2 set at such an angle that 50% of the photons are transmitted, and 50% reflected. We must do this by means of a couple of counters, and we find that each individual photon goes either into one counter or the other. The beam is split, but no single photon is ever split in this device. However, we can show that each beam is now polarized in the directions determined by P_2, simply by placing another polarizer (oriented like P_2) between P_2 and the counters and observing no change in the relative frequency of the count rates.

In quantum mechanical terms we think of the direction of polarization as an observable, and the state of polarization as determining the relative probabilities of the various possible values of the polarization observable. To measure the polarization we may place an analyzer in the beam, followed by a counter, and find the orientation of the analyzer that yields the maximum count rate. This measurement procedure never gives the polarization of a single photon, only of the beam as a whole. A single photon does not "have" a polarization.

In other words we cannot measure the polarization of any single photon, even if we have prepared a pure polarization beam. Thus, suppose we have prepared a beam pure polarized in the x-direction. Conventionally we then know that every photon in the beam is polarized in the x-direction. But consider how one would try to verify this experimentally. We would place an analyzer in the beam with its direction at an angle q to the

x-direction, followed by a counter-detector. We detect photons counted as a function of q. We must also have a counter to detect reflected (rejected) photons. The ratio of the two counts $n_t/n_r = r(q)$ must have the correct form required by theory for a pure polarized beam. The detection of any one single photon in the apparatus can never verify the polarization that photon "had" before it was passed by the analyzer and detected. In other words, even to verify the purity of the polarization, we must observe a large number of photons; we can never say anything from the observation of a single photon. Again we must emphasize that a single photon cannot be assigned a polarization by any operational definition. Polarization is a characteristics of the beam—or of the ensemble—or of the preparation procedure used to produce the ensemble.

Although this conclusion is at variance with a popular interpretation, we have been at pains above to point out that it is exactly consistent with the operational procedures used in the laboratory. The conventional neoclassical interpretation, which assigns polarization to a single photon, is a good example of the erroneous application of classical concepts (billiard ball mechanics) to quantal objects (photons). However incompatible it may be with a mechanistic world view, the correct quantal interpretation which associates states with preparations or ensembles is entirely consistent with experimental physics.

It is generally believed, on the basis of electromagnetic theory, that polarization phenomena are to be understood in terms of photon angular momentum, or spin. Thus a pure circularly polarized beam is described by an eigenstate of the angular momentum component of the photon in its propagation direction. However, the experimental operations required to *measure* angular momentum directly are entirely different from those described above for *preparing* polarization states. Let us imagine—and here we are more than ever in the domain of ideal gedankenexperiments—a detector responsive to the spin of a photon. If we place this spin-detector in a beam of photons, a spin datum will be registered for each photon. Subsequent to detection, however, it will be quite incorrect to say that photon has the spin or polarization eigenstate associated with the measured eigenvalue, for the photon will have been destroyed.

The classical Maxwellian theory will predict the averages of these spin-detector readings. In fact, coherent interference in that theory corresponds to pure quantal superposition of spin eigenstates, while an incoherent wave is now an average associated with a quantum mechanical mixture state.

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