QUANTUM THEORETICAL CONCEPTS OF MEASUREMENT: PART II*

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This portion of the essay concludes a two-part paper, Part I of which appeared in an earlier issue of this Journal. Part II begins with a careful study of the quantum description of real experiments in order to motivate a proposal that two distinct quantum theoretical measurement constructs should be recognized, both of which must be distinguished from the concept of preparation. The different epistemological roles of these concepts are compared and explained. It is then concluded that the only possible type of "quantum measurement theory" is one of little metaphysical interest and that quantum measurement seems problematical only when viewed from an overly narrow classical perspective.

10. Infinite regression. Thoughtful analysis of the standard theory of quantum measurement, or any of its variations, leads most theorists to recognize an interesting basic property of the usual approach. This property, sometimes called *infinite regression,* is received with varying degrees of enthusiasm depending on the metaphysical outlook of the critic. The essence of infinite regression is contained in this question: what performs the measurement upon $\mathscr{M}$? Ordinary measurement theory can only reply that a second apparatus $\mathscr{M}'$ must interact with $\mathscr{M}$ in the same manner $\mathscr{M}$ interacts with $\mathscr{S}$, i.e. with the effect that a measurement performed upon $\mathscr{M}'$ permits certain prediction of what a concurrent measurement on $\mathscr{M}$ would have yielded. Obviously, this suggests inquiry as to what makes measurements on $\mathscr{M}'$, and so on *ad infinitum.*

In his original formulation of standard quantum measurement theory, von Neumann did not regard infinite regression as an undesirable attribute, but rather as a necessary characteristic expressing in mathematical terms the notion of psychophysical parallelism. This idea derives from the elementary principle that all empirical observations must ultimately be regarded as perceived by the mind; the perception itself is an utterly primitive awareness of the given, a process intrinsically irreducible to scientific law. Thus, in every application of the scientific method, at some stage there must be statements to the effect that an *observer* simply observed some datum, and this will be true no matter how far into his brain the scientific analysis penetrates. Consider, for example, a measurement apparatus which registers its result as a pointer reading. It is most practical to terminate the analysis of this measurement act by saying that the observer observes the position of the needle. However, it is possible to go much further; for example, suppose the observation is made visually. An electrodynamic treatment of the relevant interactions among pointer, light, and eye can be invoked to explain the formation of a retinal image

* Received May, 1967. Part I of this article appears in the immediately preceding issue of *Philosophy of Science*, Vol. 35, No. 3 (September, 1968).

of the needle and scale; but if this work is carried out in hopes of explaining away the observer, the effort is wasted. Instead of saying the observer observed the needle, we can now say that he observed the retinal image of it, but the necessity of the observing consciousness itself is as strong as before. It should be clear that no study of the optic nerve or even of electrical properties of the brain could possibly terminate otherwise than in a statement that the observer becomes aware of the needle position, or perhaps that this awareness occurs simultaneously with some electrical effect in his brain, which would mean that a neurophysiologist studying the observer's brain would *observe*, say, a certain electroencephalogram pattern concurrently with the observer's announcement of the needle observation.

The primacy of the conscious mind in all scientific endeavor has the character of a general philosophic truth, and it is unfortunate that this lofty point was ever dragged down even as close to practical physics as quantum measurement theory. There the false impression has arisen that physics, or at least quantum physics, possesses an undesirable *subjective* element which must be reckoned with somehow.

In attempting to mathematize the subjectivistic excesses of some Copenhagen pronouncements, von Neumann therefore proposed two distinct processes, motion (P3) and measurement (P′), the latter representing the final transition to a consciousness. His motivation for drawing up the standard theory of measurement was to establish the consistency of P3 and P′ in the sense that the "cut" between observer and observed which P′ bridges should be arbitrary. Thus, in the standard theory, the same results obtain for $\mathscr{S}$ if $\mathscr{M}$ measures $\mathscr{S}$ or if $\mathscr{M}'$ measures $\mathscr{S} + \mathscr{M}$, etc.

However, we have seen earlier that this theory cannot reasonably be called *the* quantum theory of measurement. Are we therefore faced with an unusual subjective feature in quantum theory? The answer is negative, for in light of the understanding of quantum theory elaborated in foregoing sections, we deny not just the popular solution to this "quantal mind-body problem" but the problem itself. The foundations of quantum theory nowhere exhibit any more or less "subjectivism" than does classical mechanics; both theories, as has already been noted in another context, are easily accommodated by the same epistemological framework. And infinite regression is as much a property of classical as of quantum theories.

Nevertheless, von Neumann's recognition of that property and his mathematical enshrinement of it in the projection postulate has sometimes induced the belief that quantum theory carries a destructive subjectivistic quality which must be eliminated in order to save objective science. Thus justification is often sought for replacement of pure states involving apparatus by mixtures (section 9); presumably, this would halt the regression by inserting a *classical* level and therefore closing out the unwanted subjectivism. This illusion has its roots in the mechanistic philosophy widely held in the classical epoch of physics, when physical laws were regarded as purely objective "discoveries" totally divested of any metaphysical format constructed by the physicists themselves. Actually, this tenet was philosophically unacceptable [18] even in the heyday of classical physics; it is therefore strange that objectivity for quantum theory should be sought by relating it to classical physics. Both theories are subjective and objective in exactly the same ways

[9], and both display the same infinite regression property. The main point we wish to emphasize here is that this characteristic is not problematical, does not deprive science of objectivity, but rather indicates that objectivity is established within "subjective" experience.[19] However, the problem of measurement in quantum physics in the context of the present investigation is not of such philosophic depth as to require further discussion in this vein.

Accordingly, we now dismiss this basic notion of infinite regression from further consideration, since it darkens more than it illumines the problem at hand, viz. to clarify the meaning of the quantal terms measurement and preparation. Nevertheless, the *logical structure* of the infinite regression analysis does prove to be quite valuable in this connection, provided the above-mentioned efforts to link it to the mind-body problem are forgotten.

Consider again the skeletal framework of quantum measurement theory, according to which an $\mathscr{A}$-measurement on $\mathscr{S}$ consists of an interaction with an $\mathscr{A}$-meter $\mathscr{M}(\mathscr{A})$ which establishes some correlation between relevant states of $\mathscr{S}$ and $\mathscr{M}$. The $\mathscr{A}$-measurement is then carried out by observing the "reading" of $\mathscr{M}(\mathscr{A})$. Undoubtedly, this account does offer the correct quantal description of many laboratory procedures; but we now suggest that it does not deserve the name usually given it—the quantum theory of *measurement*. This "theory" cannot be said to explain the concept measurement; indeed, as we shall see below, this theory cannot even be stated carefully without implicitly using the term *measurement* itself several times. In this respect, a quantal description of a measurement process differs markedly from its classical counterpart, which does not require the term measurement at all until the final stage when an observer "looks at" the meter. We shall see below that the resultant dichotomy of meaning for the term measurement in its classical and quantum usages is traceable to the respective characters of classical and quantal observables (section **4**).

To verify our claim that so-called quantum measurement theory is not even statable without using the term measurement itself, consider its essential feature, the establishment of correlations. In classical physics, where observables may be assigned values possessively, correlations between $\mathscr{S}$ and $\mathscr{M}$ refer to these possessed physical quantities independently of measurement. This scheme, however, is inconceivable within the quantal framework, owing to the essential latency of observables. No matter what specific form correlations may take, in quantum theory they are inevitably nothing but connections among potential measurement results. For example, the correlation assumption of sec. **7**,

$$T_A(\psi \otimes \chi_0) = \sum_k \langle \alpha_k, \psi \rangle \alpha_k \otimes \gamma_k,$$

strictly implies only this: a simultaneous $\mathscr{A}$-*measurement*$_1$[20] on $\mathscr{S}$ and $\mathscr{C}$-*measurement*$_1$ on $\mathscr{M}$ at the completion of the *measurement*$_2$ interaction will yield the pair (a_k, c_l) with nonzero probability only if $k = l$. It is therefore said that the measurement process renders the $\mathscr{A}$-measurement on $\mathscr{S}$ redundant, since a $\mathscr{C}$-measurement on $\mathscr{M}$ is sufficient for prediction *with certainty* as to what the post-measure-

[19] For a further discussion on objectivity in quantum mechanics, cf. [12].

[20] Subscripts on the term *measurement* will be referred to later.

ment_2 $\mathscr{A}$-measurement_1 would yield. (It is interesting to note further that nothing can be said with certainty about what result would be obtained in an $\mathscr{A}$-measurement_1 on $\mathscr{S}$ just before the measurement_2 interaction, except that $|\langle\alpha_k, \psi\rangle|^2$ is the common probability distribution for $\mathscr{A}$-measurements_1 before and after measurement_2 interactions of this type.)

Thus we see that a rigorous quantum description of a measurement correlation process is a verbally cumbersome account in which the concept measurement itself enters repeatedly. This recurrent use of the term measurement is unavoidable in any quantal description which adheres strictly to the latent character of quantum observables. It will be urged in the concluding sections that this essential recurrence is the key to understanding the epistemological status of the quantum term measurement. To summarize: the goals of the present section have been (1) to point out that the philosophic problem of infinite regression to consciousness is equally relevant to both classical and quantum physics (and equally beyond the proper domain and competence of both); and (2) to show that close logical scrutiny of any measurement scheme in a manner suggested by the infinite regression argument (viz. posing questions like "in what sense does $\mathscr{M}$ measure $\mathscr{S}$?" and "what measures $\mathscr{M}$?") reveals that the quantum concept measurement, unlike the classical one, *must* appear as a primitive term even in the so-called quantum theory of measurement itself.

11. Quantum explanation of a real measurement. The favorite motivating experiment for measurement theorists seems to be that of Stern and Gerlach, which is alleged to be an example of the correlation $T_A(\alpha_k \otimes \chi_0) = \alpha_k \otimes \gamma_k$, and is occasionally elevated to the status of prototype for most, if not all, measurements. This view clearly exaggerates its importance; nevertheless, the Stern-Gerlach experiment is a good one to examine, if only because of its relative simplicity. We therefore present a somewhat unconventional analysis of it.

Before getting immersed in the mathematics, let us briefly recapitulate the data originally reported by Stern and Gerlach. A beam of silver atoms, emanating from a slit in a furnace, was channeled between magnetic pole pieces toward a glass plate, upon which silver deposits eventually accumulated. One pole piece was knife-edged, the other flat; hence, the silver atoms traversed an inhomogeneous magnetic field. Stern and Gerlach studied microphotographs of the deposits and interpreted what they saw as follows [5]: "The pictures show that the silver atom beam in an inhomogeneous magnetic field is split up into two beams in the direction of the inhomogeneity, one of which is attracted to the knife-edged pole and the other of which is repelled." This 1922 description is slightly tainted by classical language. A "pure" quantum theorist would interpret the same photographs this way: Position measurements on an ensemble of silver atoms, each prepared by emission from a furnace and passage through an inhomogeneous magnetic field, yield results whose statistical distribution exhibits two sharp peaks along the direction of inhomogeneity of the field. (Often there are more such peaks, but if ground state hydrogen atoms are used, as Phipps and Taylor [14] have done, there are always just two.)

To explain this phenomenon quantum mechanically, the initial state vector of the hydrogen atom[21] upon emergence from its source is assumed to be of the form $\psi \otimes \chi_0$. In the Schrödinger-Pauli representation, the spinor ψ involves only electronic coordinates relative to the nucleus, while χ_0 is a fairly localized wave packet whose argument is the atomic "center of mass." Thus the atom is formally regarded as though it were a composite system whose constituents are initially in states ψ and χ_0, a feature to be exploited later on (section **14**). Let α_1, α_2 be the eigenvectors belonging to the component of spin in the inhomogeneity direction of the magnetic field. When the temporal evolution from initial state $\psi \otimes \chi_0$, where ψ is the ground state, is calculated, the following result is obtained:

$$T(t)\,(\psi \otimes \chi_0) = \sum_k \langle \alpha_k, \psi \rangle\, \alpha_k \otimes \gamma_k(t) \equiv \Psi(t).$$

Of special interest is the center-of-mass motion, represented in the equation above by $\gamma_k(t)$, since it is the final position distribution of the atoms that the Stern-Gerlach apparatus displays. This problem is solved by examining the final center-of-mass position probability density. If δ_{XYZ} denotes a common eigenvector of center-of-mass coordinates $\mathscr{X}$, $\mathscr{Y}$, $\mathscr{Z}$, the required probability density is

$$\begin{aligned} w(X, Y, Z; \Psi(t)) &= \langle \Psi(t), 1 \otimes P_{\delta_{XYZ}}\, \Psi(t) \rangle \\ &= \left\langle \sum_l \langle \alpha_l, \psi \rangle\, \alpha_l \otimes \gamma_l, \sum_k \langle \alpha_k, \psi \rangle\, \alpha_k \otimes \langle \delta_{XYZ}, \gamma_k \rangle\, \delta_{XYZ} \right\rangle \\ &= \sum_{kl} \langle \psi, \alpha_l \rangle \langle \alpha_k, \psi \rangle\, \delta_{lk} \langle \gamma_l, \delta_{XYZ} \rangle \langle \delta_{XYZ}, \gamma_k \rangle \\ &= \sum_k |\langle \alpha_k, \psi \rangle|^2 |\langle \delta_{XYZ}, \gamma_k \rangle|^2 \end{aligned}$$

It now turns out [6] that if $\mathscr{Z}$ is the direction of field inhomogeneity, $|\langle \delta_{XYZ}, \gamma_1 \rangle|^2$ is negligibly small except in the same $\mathscr{Z}$-interval as one of the observed accumulations on the final plate; similarly, $|\langle \delta_{XYZ}, \gamma_2 \rangle|^2$ practically vanishes outside the neighborhood of the second deposit. The theory therefore fully accounts for observations of the Stern-Gerlach type described empirically above. Moreover, the theory also reveals an interesting correlation between the internal eigenstates and the center-of-mass motion. To be specific, in the expression for $w(X, Y, Z)$, the "strength" of the kth peak $|\langle \delta_{XYZ}, \gamma_k \rangle|^2$ is "weighted" by $|\langle \alpha_k, \psi \rangle|^2$, a functional of the internal eigenvector α_k. This property is often invoked in Stern-Gerlach-centered discussions on measurement theory. However, before getting into that, let us determine to what extent the quantal concept measurement has already been used in the foregoing theoretical explanation of the actual Stern-Gerlach data.

First an assumption was made about the initial state $\psi \otimes \chi_0$; this amounted to a number of conditional statements involving measurements never performed. For example, to assume the hydrogen atom is initially in its ground state means, among

[21] We take for simplicity the Phipps-Taylor case.

other things, that if energy were measured, the result would be the lowest energy level. This only illustrates that the concept of preparation ultimately depends on that of measurement. To say that a certain physical act *prepares* a state ρ always implicitly entails a set of conditional statements involving measurement in an essential way. Nevertheless, in the Stern-Gerlach experiment itself, none of these measurements relating to the preparation of $\psi \otimes \chi_0$ is performed. It may be assumed that such measurements have been made extensively in the past on a similar oven-slit device or other source and that it is guaranteed to be a bona fide producer of ensembles with $\rho = P_x \otimes \chi_0$.

The only measurements mentioned above in connection with the Stern-Gerlach experiment as if they were actually performed are of the observables X, Y, Z, i.e. atomic (center-of-mass) *position coordinates*. By contrasting this description involving atomic position measurements to the more prosaic laboratory report of Stern and Gerlach, we obtain a first indication of a point to be developed later, viz. that the quantal concept of measurement is far more abstract and less empirical than its name suggests.

The original Stern-Gerlach detection scheme—microphotographs of silver deposits—was in fact too crude to perform a single position measurement. Yet quantum theory explains the pattern of silver deposits as if they represented numerous elementary position measurements, although the adherence of a single silver atom to the glass plate is certainly never really observed. However, position measurements upon single microcosmic systems are not impossible; on the contrary, *position* is in a sense the most nearly "observable" micro-observable there is, as will become increasingly evident below. Now, suppose the glass plate is replaced by a better detector which is able to perform an operation worthy of the name position measurement. For example, impact of a single atom may trigger an "avalanche" of reactions about the collision point which produce a photographable "spot." Atomic position can then be defined operationally by equating the center coordinates on the spot with the "result of a position measurement." (These coordinates are determined by a "ruler," a macroscopic device which, used correctly, will yield the same numbers regardless of the intuitive world view of the experimenter; indeed he may employ quantum, Newtonian, or Aristotelian mechanical concepts for his own personal thoughts about rulers.) Using that rule of correspondence to relate the construct *position measurement* to empirical observation, the quantal explanation of the Stern-Gerlach effect in terms of "single position measurements" is no longer problematical; however, this has not really been the main point of this paragraph. Of more general value is the identification of *one* rule of correspondence between a quantum observable (position) and a laboratory operation.

Despite its innocent appearance, the foregoing operational definition is in experimental practice not merely a specialized example; it is rather *the* fundamental rule of correspondence in quantum physics, in the sense that all other quantum observables are actually measured by establishing correlations with the observable *position*. All measurement paraphernalia—photographic emulsions, cloud chambers, bubble chambers, counters—"directly" measure position in a manner similar to that described above. As Landé puts it,[22] "... nowhere in physics do we have 'direct' data, the only exception being location in space and time, that is (q, t)-values. Velocity, momentum, energy, etc. are always determined indirectly." DeBroglie makes the same point as follows [2]: "Any process of measurement of a dynamic variable, such as the energy and momentum of a particle, is a complex and indirect process which necessarily utilizes direct observation of particle localizations." Probably deBroglie's "necessarily" is too strong; the dominant practical role of the observable *position* is a matter of fact rather than logic; but recognition of this fact sheds more light on the nature and meaning of the quantal construct measurement than do any of the so-called "theories of measurement" reviewed in previous sections. What it suggests is that most quantum observables are never "observed," and that most of the measurements which are unavoidably mentioned in every quantum theoretical explanation are in fact never performed. Indeed, in a certain sense, they are perhaps unperformable. The remainder of this work is devoted to the amplification and clarification of these remarks.

12. Construction of an operational definition. Consider the quantum observable spin. How can it be measured? What does it mean to say that systems from an ensemble with state vector ψ will upon measurement of the Z-component of spin, $\mathscr{S}_z$ yield $\frac{1}{2}\hbar$ with relative frequency $|\langle\alpha_1, \psi\rangle|^2$? It is instructive to take a close look at how an operational definition of $\mathscr{S}_z$ is usually developed from the Stern-Gerlach experiment. Once again we suppose that an ensemble of ground state hydrogen atoms is available for study. As noted above, the Stern-Gerlach apparatus brings about this state evolution:

$$T(t)\,(\psi \otimes \chi_0) = \sum_k \langle\alpha_k, \psi\rangle\, \alpha_k \otimes \gamma_k(t).$$

We have already seen that $w(X, Y, Z)$ suggests an interesting correlation between $\mathscr{S}_z$ and $\mathscr{Z}$, due to properties of the γ_k. This may be seen more clearly by computing the joint probability for results of S_Z and Z-measurements[23] at time t. Let $\{s_k\}$ denote eigenvalues of S_Z; then the joint probability density $w(s_n, Z)$ is found as follows:

$$\begin{aligned} &w(s_n, Z)\\ &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \Big\langle \sum_l \langle\alpha_l, \psi\rangle\alpha_l \otimes \gamma_l \Big|\, P_{\alpha n} \otimes P_{\delta_{XYZ}} \,\Big|\, \sum_k \langle\alpha_k, \psi\rangle\alpha_k \otimes \gamma_k \Big\rangle\, dX\,dY\\ &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \sum_{kl} \langle\psi, \alpha_l\rangle\, \langle\alpha_k, \psi\rangle\, \langle\alpha_l, \langle\alpha_n, \alpha_k\rangle\alpha_n\rangle\, \langle\gamma_l, P_{\delta_{XYZ}}\,\gamma_k\rangle\, dX\,dY\\ &= |\langle\alpha_n, \psi\rangle|^2 \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} |\langle\delta_{XYZ}, \gamma_n\rangle|^2\, dX\,dY. \end{aligned}$$

Recall that $|\langle\delta_{XYZ}, \gamma_n\rangle|^2$ almost vanishes except near one of the Stern-Gerlach

[22] [9], 121.

[23] We now drop the notational distinction between observables $\mathscr{S}_z$, $\mathscr{Z}$ and operators S_z, Z.

accumulations, which we shall call the nth region. The distribution $w(s_n, Z)$ therefore implies that with near certainty an S_Z-measurement would yield s_n when and only when a simultaneous Z-measurement yields a result in the nth region. This leads to the common identification of the Stern-Gerlach apparatus as a kind of "spin-meter" which operates as follows: to measure the observable S_Z (on a ground state hydrogen atom), direct the atom through a magnetic field inhomogeneous in the Z direction, and then measure Z, already operationally defined. A Z-result in the nth region is considered to be a "reading" s_n of the "spin-meter."

It is therefore tempting just to regard this procedure as the empirical meaning of the quantal term spin-measurement. Unfortunately, this cannot be done for two reasons: (1) the operational definitions of Z and S_Z would then be contradictory, and (2) the Stern-Gerlach method cannot *be* a spin-measurement because its own detailed quantum mechanical description involves the concept spin-measurement in a logically anterior way.

Reason (1) is a consequence of the mathematical fact that γ_1, γ_2 are not quite orthogonal. Thus, although $|\langle \delta_{XYZ}, \gamma_n \rangle|^2$ is minuscule outside the nth region, *it does not vanish*. Hence, there is a finite probability, for example, that simultaneous Z- and S_Z-measurements would yield a Z-result in region 1 and the S_Z-eigenvalue s_2. In other words, we are able to evaluate "how good" a "spin-meter" the Stern-Gerlach device is; therefore it cannot be used to define the quantal term spin-measurement. If it were so employed, the operational definition of Z would be contradicted: the appearance of a "spot" in the nth region would always mean that an S_Z-measurement has yielded s_n but would no longer indicate with certainty that the Z-measurement result coincided with the "spot"! The best conclusion seems to be that the Stern-Gerlach "spin-meter" is excellent but not perfect, and hence unsuitable for defining the concept of spin-measurement. The importance of this result lies in the fact that in practice the above "spin-meter" seems to be the only kind there is; therefore, the construct spin-measurement—of proven value in theoretical explanations—refers to no actual "laboratory measurement" at all. This suggests perhaps that quantum physics uses the term measurement in two distinct senses, one traditional and one peculiarly quantal. That such is the case will emerge presently from the following consideration of reason (2).

Since it is universally applicable to any conceivable quantal description of a "laboratory measurement," reason (2) is more fundamental than reason (1). The principal point has already been discussed in some generality at the end of section **10**: the term measurement *necessarily* occurs as a primitive even in a quantal description of a measurement process. In the present case, this means that a *careful* account of the operation of a Stern-Gerlach "spin-meter" runs as follows: if an S_Z-measurement$_1$[24] on the atom just prior to the measurement$_2$ interaction with the "spin-meter" would certainly have yielded s_n, then immediately after the measurement$_2$ interaction, an S_Z-measurement$_1$ will certainly yield s_n and a Z-measurement$_1$ will (almost) certainly yield a Z-value in the nth region. Hence the post-measurement$_2$ S_Z-measurement$_1$ is redundant; a post-measurement$_2$ Z-measurement$_1$ is sufficient to deduce what an S_Z-measurement$_1$ would have given

[24] As before, the subscripts should be ignored for the moment.

at the instant the Stern-Gerlach measurement$_2$ procedure began. We hasten to point out that *this is not mere semantic legerdemain. No alternative quantal description is conceivable*; to explain in detail the operation of a Stern-Gerlach "spin-meter" in any other way is impossible within the language of quantum theory! Reference *must* be made to the imaginary results of S_Z-measurements which are never performed in any laboratory. Furthermore, the very concept of performing an S_Z-measurement seems to designate no empirical act whatsoever. Thus the Stern-Gerlach device is said to reveal "what an S_Z-measurement would have given" earlier (just before the atom entered the magnetic field); yet this earlier S_Z-measurement itself is not even an imaginable laboratory operation. Indeed, the very device which supposedly performs that S_Z-measurement can itself be described only in terms of what the S_Z-result would have been if S_Z had been measured! As suggested in section **14**, similar conclusions may be drawn from the quantal explanation of any "laboratory measurement" procedure whatever. An experimental scheme designed to "make measurements of $\mathscr{A}$" will in general be described in terms of unperformed and unperformable $\mathscr{A}$-measurements.

13. The dual meaning of measurement in quantum physics. Imagine for a moment that S_Z were an observable in the classical sense; it would then be meaningful to say that the atom entering the Stern-Gerlach device *has* $S_Z = s_n$. Assume further that theoretical analysis demonstrates, in analogy to the quantal case, that such an atom is always channeled to the nth region. It would then be permissible to conclude that the term S_Z-measurement merely refers to this operation: pass the atom through a Stern-Gerlach device and observe the spatial region of its emergence. This act is a measurement in the classical sense because it leads to a determination of what S_Z-value the atom *possessed.* Furthermore, nowhere in this or any classical description of a measurement procedure does the term measurement itself enter in a fundamental way.

By contrasting this fictitious classical description of a Stern-Gerlach "spin-meter" to its quantal counterpart, the source of difficulty in the quantum case becomes apparent. At several points in the classical account of a measurement process, the concept of possessed observable is employed; but at the analogous places in a quantal account, this notion cannot be used. The basic structure of quantum theory forbids it. In section **4**, we noted that this old concept of possession had been superseded in quantum theory by the idea of *latency.* Thus the quantum axioms (section **2**) connect observables to systems and states only in a dispositional sense. This connection is made through the primitive construct $\mathscr{A}$-measurement, about which nothing is said except that when it is performed upon a system, it yields a number. Hence the logically primitive construct $\mathscr{A}$-measurement, a consequence of latency, plays a role in quantum theory analogous to that of possession in classical theory. Accordingly, to convert a classical description of a measurement procedure to a quantum description, each classical statement of the form, "$\mathscr{S}$ has $\mathscr{A} = a_k$" must be replaced by the quantal proposition, "an $\mathscr{A}$-*measurement* upon $\mathscr{S}$ would yield a_k."

The term *measurement*, as it appears in the quantum axioms, has therefore a theoretical status quite distinct from that of the term *measurement* in its classical usage and in the phrase, "theory of measurement." In recognition of his homonymy, we shall henceforth designate the primitive construct measurement, which is essential to the statement of quantum axioms, as $\mathscr{M}_1$; an $\mathscr{A}$-measurement will be denoted by $\mathscr{M}_1(\mathscr{A})$. On the other hand, $\mathscr{M}_2(\mathscr{A})$ will represent the classical concept of measurement, or at least the nearest quantal analogue to it. For example, the fictitious classical $\mathscr{M}_2(S_Z)$ is an operation which employs physical interaction to establish a correlation between possessed S_Z- and Z-values of the atom, so that an observation of the Z-value (i.e. intelligently "looking at" the "spot" and "ruler") enables inference of the S_Z-value. Similarly, the quantal $\mathscr{M}_2(S_Z)$ is an operation which employs physical interaction to establish a correlation between the potential results of $\mathscr{M}_1(S_Z)$ and $\mathscr{M}_1(Z)$ so that the actual result of a performed $\mathscr{M}_1(Z)$ enables inference of the potential result of a never performed $\mathscr{M}_1(S_Z)$. This important distinction between $\mathscr{M}_1$ and $\mathscr{M}_2$ was hinted at in sections **10** and **12**, where subscripts were attached to the word measurement to suggest its two meanings in quantum parlance.

In connection with $\mathscr{M}_1$ and $\mathscr{M}_2$, several questions must be raised: (1) How does $\mathscr{M}_1$ fit into the general epistemological framework of physics? (2) Similarly, what is the role of $\mathscr{M}_2$ in the scientific method? (3) How are $\mathscr{M}_1$ and $\mathscr{M}_2$ related? and (4) What has all this to do with the quantum theory of measurement? We shall now discuss these points in that order.

(1) The most striking property of $\mathscr{M}_1$ is its abstractness; it is an ultimate primitive construct irreducible to any others. Epistemologically it is like the concepts physical quantity and mass point in classical mechanics—no phenomenon can be theoretically comprehended without it. Yet in spite of its deeply theoretical status, the nature of $\mathscr{M}_1$, as implied by its role in the quantum axioms, suggests an abstract mimicry of a naive view which equates measurement and elementary observation. Thus the statement—"if $\mathscr{M}_1(\mathscr{A})$ is performed upon $\mathscr{S}$, it will yield the result a"—is beguilingly similar in form to this: "if an observation is made of the sky, it will 'yield' the color blue." However, the apparent similarity is purely grammatical; the differences are far more important. If a literal interpretation is demanded for the clause "$\mathscr{M}_1(\mathscr{A})$ is performed upon $\mathscr{S}$," then we shall have to provide some kind of "microelf," or "quantum demon," to do the performing; for, as we have just seen in the special case of spin-measurements, real physicists do not, indeed cannot, "perform $\mathscr{M}_1(S_Z)$." $\mathscr{M}_1(S_Z)$ is quite typical in this respect. Hardly any $\mathscr{M}_1(\mathscr{A})$ is ever "performed" by an experimenter; in practice, macro-position observations are perhaps the only exception. Nevertheless, the construct $\mathscr{M}_1(\mathscr{A})$, even if it is imagined to represent the perceptions of omnipresent q-demons, is invaluable and unavoidable in the quantum theoretical explanation of all actual empirical observations.

(2) In the remarks of section **3**, the term *measurement* was tacitly interpreted in the usual way as the epistemological link between percepts and concepts. On reflection we now see that this ordinary measurement concept is $\mathscr{M}_2$. By implicitly using $\mathscr{M}_2$ to discuss the term *measurement* in the quantum axioms, we confused $\mathscr{M}_2$ in the natural manner with $\mathscr{M}_1$, which we now recognize as the only measure-

ment-construct appearing in those postulates. However, the fundamental mediatory role of $\mathcal{M}_2$ is the same in quantum physics as in the remainder of science; the novelty of the quantum framework lies in the fact that, as a consequence of the latency of observables, among the constructs which $\mathcal{M}_2$ relates to datal experience is $\mathcal{M}_1$.

(3) In classical physics, an $\mathcal{M}_2(\mathcal{A})$ procedure is always understood as establishing a correlation between the possessed value of an abstract $\mathcal{A}$ and the possessed value of some observable $\mathcal{X}$ *directly* accessible to the experimenter. With de Broglie and Landé, we have tentatively (cf. section **15**) adopted the view that $\mathcal{X}$ is always essentially a *position*; thus the physicist always "looks at" a "spot" and "ruler," thereby observing $\mathcal{X}$ directly. It should be clear that without the latter direct observation, $\mathcal{M}_2(\mathcal{A})$ would be impossible and the theory at hand therefore physically meaningless. Applied to the quantum case, this means that there must exist an $\mathcal{X}$ directly observable by a real experimenter; i.e. the physicist himself must be the q-demon who performs $\mathcal{M}_1(\mathcal{X})$ by a simple "look-and-see" observation. As before, $\mathcal{X}$ is presumably a *position*, which the physicist observes directly as a coincidence of "spot" and "ruler." Were he a quantum purist, he would of course describe his actions as follows: "$\mathcal{M}_1(\mathcal{X})$ was performed ('ruler' placed near 'spot') and yielded x ('spot' coincided with 'ruler' mark x)." Thus for $\mathcal{M}_2$ to be possible at all, a quantum physicist must for *some* $\mathcal{A}$ be himself a q-demon capable of "performing" $\mathcal{M}_1(\mathcal{A})$, although for most $\mathcal{A}$'s his "performing $\mathcal{M}_1(\mathcal{A})$" is as inconceivable as, for example, the direct perception of (possessed) energy by a classical physicist.

(4) Having established the twofold meaning of measurement in quantum physics, we are now able to state precisely the very most that any so-called quantum theory of measurement could hope to explain. Simply put, such a theory can only offer a description of an *$\mathcal{M}_2(\mathcal{A})$ in terms of $\mathcal{M}_1(\mathcal{A})$ and other $\mathcal{M}_1$'s*. To reach that conclusion, we assume that the final purpose of formulating a "quantum measurement theory" would be to give a quantal description of actual laboratory measurement processes, in particular, to achieve a quantum theoretical understanding of how information about the microcosm is obtained. Roughly speaking, it is obvious that knowledge of things unperceivable must be gained through correlations with things directly apprehended. To be more scientific, a microsystem can be studied only via *physical interaction* with it; otherwise, the requisite correlations could not be established. On the other hand, a laboratory measurement scheme $\mathcal{M}_2$ can be exhaustively described without using any postulates except those normally required to explain other physical processes; this fact was essentially the crux of previous sections in which we criticized the various extant ideas about measurement, some of which seemed to regard quantum measurement processes as more than just physical processes. Therefore, an $\mathcal{M}_2$ can and must always be explained in terms of the basic constructs of quantum theory, among which are the $\mathcal{M}_1$'s. The $\mathcal{M}_1$'s themselves, being ultimate primitive constructs, are not susceptive of further quantal explanation. A "quantum theory of $\mathcal{M}_1$" would be tautological, like a "mechanical theory of motion." Hence, a quantum theory of measurement can at most be a quantum theory of $\mathcal{M}_2$.

14. Remarks on preparation. Several times in previous sections we have alluded to the interdependence of the concepts measurement and preparation. Unfortunately, many treatments of "measurement theory" fail to stress the *differences* between these concepts. Frequently, measurement and preparation are regarded as essentially equivalent; this premise leads inevitably to "measurement" discussions marked by severe ambiguities. For example, Schwinger's "algebra of measurement" [17] is really a hybrid "algebra of measurement and preparation" in which the two concepts are not carefully distinguished; and Groenewold [7] overtly ignores the difference: "I take all the time the term 'measurement' in a broad sense, including initial preparations, intermediate observations and final detections. Those who prefer another terminology are free to make the translation."

This belief that measurement and preparation are practically equivalent arises upon adoption at least of P′ as a universl quantum postulate. The latter has already been discussed briefly, the principal conclusion being that P′, although occasionally derivable, is required *a priori* in the quantal account of no phenomenon whatsoever. However, the equivalent treatment of measurement and preparation is most often founded upon *P* (the logically untenable predecessor of P′) which assigns a state vector to a single system on the basis of a measurement result. Thus it is sometimes asserted that an $\mathscr{A}$-measurement which yields a_k prepares the state α_k.

In this context, it is difficult to say whether the $\mathscr{A}$-measurement is $\mathscr{M}_1(\mathscr{A})$ or $\mathscr{M}_2(\mathscr{A})$. Very formal treatments sometimes give the impression that an $\mathscr{M}_1(\mathscr{A})$ yielding a_k is equivalent to the preparation $\Pi(P_{\alpha_k})$ of the state α_k. Others seem to suggest that when $\mathscr{M}_2(\mathscr{A})$ reveals the result a_k of an $\mathscr{M}_1(\mathscr{A})$, $\Pi(P_{\alpha_k})$ may be regarded as having occurred at the time of $\mathscr{M}_1(\mathscr{A})$, or even, as sometimes claimed [8], at any time between $\mathscr{M}_1(\mathscr{A})$ and the completion of $\mathscr{M}_2(\mathscr{A})$! In our opinion such considerations are as nonsensical as the following sentence: "a q-demon prepares $\mathscr{S}$ in the state α_k by 'looking at' the $\mathscr{A}$-ness of $\mathscr{S}$ and 'seeing' a_k."

A similar comment applies to the contention that a commitment regarding the post-measurement state is essential for the theoretical analysis of successive measurements. If "successive measurements" means successive $\mathscr{M}_1$'s, then the problem is unphysical; it amounts to an inquiry about a "sequence of q-demonic acts." On the other hand, if successive $\mathscr{M}_2$'s are contemplated, all that is involved is a physical process, fully describable without attributing any properties to successive $\mathscr{M}_1$'s.

Like $\mathscr{M}_2(\mathscr{A})$, $\Pi(P_{\alpha_k})$, or in general $\Pi(\rho)$, is correctly interpreted as a physical process which always has a quantum theoretical explanation. To illustrate this point and to clarify the distinction between measurement and preparation, we shall present briefly the careful quantal description of a preparation scheme, a quantum theory of preparation. Since it is commonly mentioned erroneously as evidence that measurement and preparation are the same, the Stern-Gerlach "spin-meter" as a preparation device will here be used to prove the opposite—that measurement and preparation are basically different.

Once again, for simplicity ground state hydrogen atoms will be fed into the magnetic field. This statement, as noted earlier, assumes that a preparation scheme for the initial ensemble of atoms is given. A theory of preparation can at most be a quantal description of the physical process by which a desired ensemble is transformed and/or extracted from an initial ensemble. It is of course impossible to describe the ensembles or the process without the primitive measurement construct $\mathscr{M}_1$. Only in this sense is preparation ultimately reducible to the concept measurement, but this is not the same as saying that measurement and preparation processes ($\mathscr{M}_2$ and Π) are equivalent.

If all members of the ensemble consisting of atoms which passed through a Stern-Gerlach field are considered, the device has obviously prepared the state,

$$T(t)\,(\psi \otimes \chi_0) = \sum_k \langle \alpha_k, \chi \rangle \alpha_k \otimes \gamma_k(t).$$

However, since the position detector is absent, the preparation is not connected with any measurement operation—a simple illustration that Π and $\mathscr{M}_2$ are not equivalent.

More interesting preparations based on the Stern-Gerlach experiment require the concepts of subensemble selection and A-equivalence. For definiteness, we consider $\Pi(P_{\alpha_1 \otimes \gamma_1})$, a preparation process often associated with the Stern-Gerlach setup. The customary description of the method is rather naive: since the magnet has spatially separated the "beam" into the two disjoint regions, an $\alpha_1 \otimes \gamma_1$-"filter" may supposedly be constructed by erecting an absorber $\mathscr{W}$ in region 2. The complete apparatus—$\Pi(P_{\Psi \otimes \chi_0})$-device, Stern-Gerlach magnet, and $\mathscr{W}$—would therefore constitute a $\Pi(P_{\chi_1 \otimes \gamma_1})$-device. Now, it may be that this experimental arrangement does indeed effect $\Pi(P_{\alpha_1 \otimes \gamma_1})$; but, if so, there must exist an explanation better than the preceding "filtration" argument. This notion of filtering arises from an erroneous classical interpretation of the aforementioned probability distribution $w(s_n, Z)$. As has been stressed repeatedly, quantum theory involves only probabilities that a specified measurement will yield a given result; it does not and cannot meaningfully speak of the probability that a system will be found "in" a given state. Thus the fact that $w(s_1, Z)$ practically vanishes outside region 1 does *not* mean that an atom detected in that region was "really in" the state $\alpha_1 \otimes \gamma_1$ all along, an obvious presupposition behind the above "filtration" scheme. The ensemble to be "filtered" is in fact pure—its density operator is the projection $P_{\Sigma_k \langle \alpha_k, \psi \rangle \alpha_k \otimes \gamma_k}$—and therefore *in principle* irreducible to distinct pure subensembles such as $P_{\alpha_1 \otimes \gamma_1}$.

This invites the possible reply in defense of the "filtration" picture that we have been ignoring the very practical fact that only a restricted set of observables A is really of any interest here. Thus our admonitions regarding the theoretic impossibility of subensemble selection, or filtration (relative to *all* observables) may be irrelevant if just the observable set A is studied. In terms of A-equivalence, it may be that

$$P_{\Sigma_k \langle \alpha_k, \psi \rangle \alpha_k \otimes \gamma_k} \underset{\sim}{\mathsf{A}} \sum_k |\langle \alpha_k, \chi \rangle|^2 \, P_{\alpha_k \otimes \gamma_k},$$

hence that the desired selection may actually be possible relative to A. As a matter of fact, if A is the set of all observables either of the form $A \otimes 1$ or $1 \otimes B$, then the foregoing A-equivalence relation is valid (to an excellent approximation).[25] To prove that, recall that the atom is formally treated as a composite system in the sense that its "internal motion" is separated from its "center-of-mass motion"; thus the desired A-equivalence may be established by proving the following relations:

(1) $\mathrm{Tr}_1\, \rho = \mathrm{Tr}_1\, \rho_M,$

(2) $\mathrm{Tr}_2\, \rho = \mathrm{Tr}_2\, \rho_M,$

where $\rho = P_{\Sigma k \langle \alpha_k, \psi \rangle \alpha_k \otimes \gamma_k}, \qquad \rho_M = \sum_k |\langle \alpha_k, \psi \rangle|^2 P_{\alpha_k \otimes \gamma_k}.$

The required calculations are straightforward:

$$\begin{aligned}
\mathrm{Tr}_1\, \rho &= \sum_n \Big\langle \alpha_n \Big| \sum_l \langle \alpha_l, \psi \rangle \alpha_l \otimes \gamma_l \Big\rangle \Big\langle \sum_k \langle \alpha_k, \psi \rangle \alpha_k \otimes \gamma_k \ \Big| \ \alpha_n \Big\rangle \\
&= \sum_{nlk} \delta_{ln} \langle \alpha_l, \psi \rangle \delta_{kn} \langle \psi, \alpha_k \rangle |\gamma_l\rangle \langle \gamma_k| \\
&= \sum_k |\langle \alpha_k, \psi \rangle|^2 P_{\gamma_k}.
\end{aligned}$$

$$\begin{aligned}
\mathrm{Tr}_1\, \rho_M &= \sum_n \Big\langle \alpha_n \Big| \sum_k |\langle \alpha_k, \psi \rangle|^2 P_{\alpha_k} \otimes P_{\gamma_k} \Big| \alpha_n \Big\rangle \\
&= \sum_k |\langle \alpha_k, \psi \rangle|^2 \Big(\sum_n \langle \alpha_n, P_{\alpha_k} \alpha_n \rangle \Big) P_{\gamma_k} \\
&= \sum_k |\langle \alpha_k, \psi \rangle|^2 P_{\gamma_k}.
\end{aligned}$$

$$\begin{aligned}
\mathrm{Tr}_2\, \rho &= \iiint dX\, dY\, dZ \Big\langle \delta_{XYZ} \Big| \sum_l \langle \alpha_l, \psi \rangle \alpha_l \otimes \gamma_l \Big\rangle \Big\langle \sum_k \langle \alpha_k, \psi \rangle \alpha_k \otimes \gamma_k \Big| \delta_{XYZ} \Big\rangle \\
&= \sum_{kl} \iiint dX\, dY\, dZ \langle \alpha_l, \psi \rangle \ \langle \psi, \alpha_k \rangle \ |\alpha_l\rangle \ \langle \alpha_k| \ \langle \delta_{XYZ}, \gamma_l \rangle \ \langle \gamma_k, \delta_{XYZ} \rangle \\
&\cong \sum_k |\langle \alpha_k, \psi \rangle|^2 P_{\alpha_k},
\end{aligned}$$

since $\langle \delta_{XYZ}, \gamma_l \rangle$, $\langle \delta_{XYZ}, \gamma_k \rangle k \neq l$, are each nonzero in disjoint X, Y, Z-regions.

$$\begin{aligned}
\mathrm{Tr}_2\, \rho_M &= \iiint dX\, dY\, dZ \Big\langle \delta_{XYZ} \ \Big| \sum_k |\langle \alpha_k, \psi \rangle|^2 P_{\alpha_k} \otimes P_{\gamma_k} \Big| \ \delta_{XYZ} \Big\rangle \\
&= \sum_k |\langle \alpha_k, \psi \rangle|^2 \Big(\iiint dX\, dY\, dZ \langle \delta_{XYZ}, P_{\gamma_k} \delta_{XYZ} \rangle \Big) P_{\alpha_k} \\
&= \sum_k |\langle \alpha_k, \psi \rangle|^2 P_{\alpha_k}.
\end{aligned}$$

[25] This equivalence class is not complete; i.e. there do exist operators outside A for which the two ρ's are equivalent. Gottfried [6], e.g. uses in the Stern-Gerlach problem a class different from our A.

It is therefore tempting to declare that the total ensemble prepared by a Stern-Gerlach device may be split into pure subensembles with state vectors $\alpha_k \otimes \gamma_k$, $k = 1, 2$, provided only observables in A are considered. Hence selection of subensemble $P_{\alpha_1 \otimes \gamma_1}$ would constitute $\Pi(P_{\alpha_1 \otimes \gamma_1};\ \mathsf{A})$ i.e. preparation of the state $\alpha_1 \otimes \gamma_1$ relative[25] to A. If this conclusion were correct, then our above critique of the "filtration" argument would be reduced in *practical* cases almost to verbal quibbling; however, owing to an inherent weakness of the "relative preparation" concept $\Pi(P_{\alpha_1 \otimes \gamma_1};\ \mathsf{A})$, we still insist that the "filtration" argument does not justify the claim that the apparatus in question—$\psi \otimes \chi_0$-source, Stern-Gerlach magnet, and $\mathscr{W}$—prepares $P_{\alpha_1 \otimes \gamma_1}$ in any sense. The trouble with "relative preparation" $\Pi(\rho;\ \mathsf{A})$ is its reliance upon A-equivalence, which is not a temporally invariant property. Although it is true, as demonstrated above, that $\mathrm{Tr}(\rho A) = \mathrm{Tr}(\rho_M A)$, for each A in A at a given instant, say t_1, it does not follow that $\mathrm{Tr}(T\rho T^\dagger A) = \mathrm{Tr}(T\rho_M T^\dagger A)$ for each A in A, where $T \equiv T(t_2, t_1)$. Consider, for example, the probability densities $w(s_n, X, Y, Z;\ \rho(t))$ and $w(s_n, X, Y, Z;\ \rho_M(t))$

$$w(s_n, X, Y, Z;\ \rho(t)) = \mathrm{Tr}(\rho(t)\, P_{\alpha_n \otimes \delta_{XYZ}}) = \Big|\sum_k \langle \alpha_k, \psi\rangle \langle \alpha_n \otimes \delta_{XYZ}, T(\alpha_k \otimes \gamma_k)\rangle\Big|^2,$$

$$w(s_n, X, Y, Z;\ \rho_M(t)) = \mathrm{Tr}(\rho_M(t)\, P_{\alpha_n \otimes \delta_{XYZ}}) = \sum_k |\langle \alpha_k, \psi\rangle \langle \alpha_n \otimes \delta_{XYZ}, T(\alpha_k \otimes \gamma_k)\rangle|^2.$$

At $t = t_1$, $T = 1$, and these expressions are of course equal; but for $t > t_1$, $T \neq 1$ and the distributions are unequal. This demonstrates that the A-equivalence of ρ and ρ_M at t_1 is useful only in statics; in general, even if the only observables ever considered are in A, still ρ cannot be replaced by ρ_M for *dynamic* applications. Only ρ correctly represents the idea of state in its causal role; ρ_M, while equivalent to ρ at t_1, leads to incorrect predictions and cannot therefore be regarded as the state. Hence the foregoing "filtration" argument cannot be accepted as a quantal explanation of the *state* preparation $\Pi(P_{\alpha_1 \otimes \gamma_1})$.

Thus far, it has been left undecided whether or not the combination of $\psi \otimes \chi_0$-source, Stern-Gerlach magnet, and $\mathscr{W}$ may actually be used to prepare $P_{\alpha_1 \otimes \gamma_1}$; all that has been established is the classical naivete of the common description in terms of "filtration." The success or failure of the proposed device as a $P_{\alpha_1 \otimes \gamma_1}$-preparer depends mainly upon the nature of the system $\mathscr{W}$. The $\psi \otimes \chi_0$-source plus Stern-Gerlach magnet has prepared $P_{\Sigma k \langle \alpha_k, \psi\rangle \alpha_k \otimes \gamma_k}$; what must be shown is that the interaction with $\mathscr{W}$ could convert this pure ensemble into a mixture from which the pure subensemble $P_{\alpha_1 \otimes \gamma_1}$ might be extracted. We shall now discuss two model theories of preparation which would explain in a sensible quantum theoretic way how the combined apparatus in question could perform $\Pi(P_{\alpha_1 \otimes \gamma_1})$.

(1) For a simple but rather fanciful model, assume $\mathscr{W}$ is a slab of antimatter initially containing N antiatoms. Since annihilation processes connected with matter-antimatter interaction must be considered, we need three Hilbert spaces:

[25] This should not be confused with Everett's concept of "relative state" [4].

$\mathscr{H}_{\mathscr{S}}$, associated with the atom; $\mathscr{H}_{\mathscr{W}}$, with the slab; and $\mathscr{H}_{\mathscr{F}}$, with electromagnetic radiation. Let φ_0 denote the radiation vacuum state, φ some other radiation state, ω_N a stationary state for the N antiatoms, ν_{N-1} some state for (N-1) antiatoms, and Ψ_0 the atom's vacuum state. Suppose the initial state of the composite system $\mathscr{S} + \mathscr{W} + \mathscr{F}$ is

$$\Big(\sum_k \langle \alpha_k, \psi\rangle \alpha_k \otimes \gamma_k\Big) \otimes \omega_N \otimes \varphi_0.$$

If U denotes the evolution operator for this total system from the preparation of the above initial state until a time when the atomic wave packet would have passed beyond the absorber position if $\mathscr{W}$ were absent, then in terms of U-matrix elements, the following assumptions define a $\mathscr{W}$ capable of producing the desired mixtures:

$$\langle \kappa \otimes \lambda \otimes \xi |\, U\, |(\alpha_1 \otimes \gamma_1) \otimes \omega_N \otimes \varphi_0\rangle = 0$$

unless $\kappa = T(\alpha_1 \otimes \gamma_1)$, $\lambda = \omega_N$, $\xi = \varphi_0$, where T describes the evolution of the atom when $\mathscr{W}$ is absent.

$$\langle \kappa \otimes \lambda \otimes \xi |\, U\, |(\alpha_2 \otimes \gamma_2) \otimes \omega_N \otimes \varphi_0\rangle = 0,$$

unless $\kappa = \Psi_0$, $\lambda = \nu_{N-1}$. (Classically speaking, these expressions mean that an atom may traverse region 1 undisturbed but in region 2 would be destroyed.)

The final state of the composite system would therefore be of the form

$$U\Big[\Big(\sum_k \langle \alpha_k, \psi\rangle \alpha_k \otimes \gamma_k\Big) \otimes \omega_N \otimes \varphi_0\Big]$$
$$= \langle \alpha_1, \psi\rangle [T(\alpha_1 \otimes \gamma_1)] \otimes \omega_N \otimes \varphi_0 + \langle \alpha_2, \psi\rangle \Psi_0 \otimes \nu_{N-1} \otimes \varphi.$$

"Tracing out" the antimatter and radiation parts, we find[27] that the atom-ensemble now has the density operator

$$\rho_1 = |\langle \alpha_1, \psi\rangle|^2\, P_{T(\alpha_1 \otimes \gamma_1)} + |\langle \alpha_2, \psi\rangle|^2\, P_{\Psi_0},$$

which shows that only the fraction $|\langle \alpha_1, \psi\rangle|^2$ of the atoms from the original $\psi \otimes \chi_0$-source emerge from the complete apparatus and that this subensemble has state vector $T(\alpha_1 \otimes \gamma_1)$. (For the short time interval of interest, $T \cong 1$; hence we have effectively a $\Pi(P_{\alpha_1 \otimes \gamma_1})$-device.)

(2) A somewhat more realistic model results if $\mathscr{W}$ is taken as a slab of ordinary matter. If U again denotes the evolution operator during interaction, the idea of "region 2 absorber" may perhaps be expressed as follows:

$$U[(\alpha_1, \otimes \gamma_1) \otimes \omega_N] = T(\alpha_1 \otimes \gamma_1) \otimes \omega_N \cong (\alpha_1 \otimes \gamma_1) \otimes \omega_N,$$
$$U[(\alpha_2 \otimes \gamma_2) \otimes \omega_N] = \sum_{lm} v_{lm} \varphi_l \otimes \omega_m,$$

where $\{\omega_k\}$ is a complete set of stationary states for $\mathscr{W}$, $\{\varphi_l\}$ is a complete eigenvector set associated with the atom, and v_{lm} has these properties: $v_{lN} \cong 0$, for each

[27] This tracing is easily done if one just notes that $\langle T(\alpha_1 \otimes \gamma_1), \Psi_0\rangle = \langle \omega_N, \nu_{N-1}\rangle = \langle \varphi_0, \varphi\rangle = 0$; the general mathematical form is then the same as that encountered in the correlation assumption of orthodox measurement theory (section 7).

l; $v_{lm} \cong 0$, unless $\sum_n |\langle \alpha_n \otimes \delta_{XYZ}, \varphi_l \rangle|^2$ is essentially nonzero only within the slab, for each m.

The final state of the total system is then

$$U[(\sum_k \langle \alpha_k, \psi \rangle \alpha_k \otimes \gamma_k) \otimes \omega_N]$$
$$\cong \langle \alpha_1, \psi \rangle (\alpha_1 \otimes \gamma_1) \otimes \omega_N + \langle \alpha_2, \psi \rangle \sum v_{lm} \varphi_l \otimes \omega_m.$$

Let $$\sum_l v_{lm} \varphi_l = g_m \theta_m, \qquad \langle \theta_m, \theta_m \rangle = 1, \quad m \neq N;$$

$$\langle \alpha_1, \psi \rangle = q_N, \qquad \alpha_1 \otimes \gamma_1 = \theta_N; \quad \text{and } \langle \alpha_2, \psi \rangle g_m = q_m, \; m \neq N.$$

With these substitutions, the final state becomes

$$\sum_k q_k \, \theta_k \otimes \omega_k$$

where $\{\omega_k\}$ is an orthogonal set but $\{\theta_k\}$ is not. Now, consider the reduced density operator for the atom alone:

$$\begin{aligned}
\rho_1 &= \mathrm{Tr}_{\mathscr{W}} \, P_{\Sigma_k q_k \theta_k \otimes \omega_k} \\
&= \sum_m \Big\langle \omega_m \Big| \sum_l q_l \theta_l \otimes \omega_l \Big\rangle \Big\langle \sum_k q_k \theta_k \otimes \omega_k \Big| \omega_m \Big\rangle \\
&= \sum_{klm} q_l q_k^* \, |\theta_l\rangle \langle \theta_k| \, \delta_{lm} \, \delta_{km} \\
&= \sum_k |q_k|^2 \, P_{\theta_k} = |\langle \alpha_1, \psi \rangle|^2 \, P_{\alpha_1 \otimes \gamma_1} + \sum_{k \neq N} |q_k|^2 \, P_{\theta_k}.
\end{aligned}$$

Thus the atom ensemble has become a genuine mixture from which it is possible to select the subensemble $P_{\alpha_1 \otimes \gamma_1}$. Since all other subensembles are localized in the slab by the interaction, we may conclude that the ensemble of atoms emerging from the complete apparatus (the fraction $|\langle \alpha_1, \psi \rangle|^2$ of the original atoms) has state vector $\alpha_1 \otimes \gamma_1$. This model therefore exemplifies a rational quantum mechanical explanation of a $\Pi(P_{\alpha_1 \otimes \gamma_1})$-device; yet no measurement$_2$ process was involved.

It should now be clear that $\Pi(P_{\alpha_1 \otimes \gamma_1})$ is a physical process different from $\mathscr{M}_2(S_Z)$ but inexplicable without tacit reference to $\mathscr{M}_1$'s. In particular, there is no justification for any general statement that $\Pi(P_{\alpha_1 \otimes \gamma_1})$ is equivalent to an $\mathscr{M}_2(S_Z)$ yielding s_1. The apparatus just described—Stern-Gerlach magnet plus region 2 absorber—would effect $\Pi(P_{\alpha_1 \otimes \gamma_1})$; but it would not perform $\mathscr{M}_2(S_Z)$ (unless the detection of *nothing* in region 1 is regarded as a measurement of S_Z yielding s_1!). Indeed the *preparation* of the $P_{\alpha_1 \otimes \gamma_1}$-ensemble still occurs even if the absorber is not a detector at all, i.e. even if it simply does not record whether or not it captured any atom.

The Stern-Gerlach example of the past few sections has demonstrated that the constructs $\mathscr{M}_1$, $\mathscr{M}_2$, and Π should be carefully distinguished in "measurement theories"; otherwise ambiguity and confusion are inevitable. An excellent example of this confusion is the term "selective measurement" [17] which is sometimes used quite indiscriminately to refer to both measurement and preparation whenever a "separation and filtration" scheme like the Stern-Gerlach device is the physical basis of both Π and $\mathscr{M}_2$. As a result, it is virtually impossible to determine the meaning or purpose of a discussion on so-called "selective measurements." The distinction between $\mathscr{M}_1$, $\mathscr{M}_2$, and Π is abnormally subtle in the Stern-Gerlach, or "selective," case; for this reason, it is a favorite example among proponents of wave packet reduction and/or the equivalence of $\mathscr{M}_2(\mathscr{A})$ and $\Pi(P_{\alpha_k})$. Nevertheless, as we have seen, the differences among these concepts can still be exposed.

Ordinary applications of quantum theory are normally successful in spite of the occasional confusion of preparation and measurement. However, the distinction can be quite important in basic theoretical considerations. A good example of faulty reasoning due to the implicit assumption that preparation must be accomplished through measurement occurs in connection with the superselection rules of quantum field theory. These rules arise from invariance principles which, applied to states, require that certain distinct state vectors (rays) be physically equivalent. Now, there are at least two ways to guarantee this equivalence: (1) Postulate that not all Hilbert vectors represent physically realizable states, or equivalently that there is no process $\Pi(P_\psi)$ for certain ψ's. The Pauli principle is a familiar example of such a requirement. In the case of superselection rules, it turns out that the distinct state vectors which must be equivalent can be eliminated by postulating that all physically realizable state vectors are eigenvectors of certain operators (total charge, e.g.). (2) Modify the common axiom that all Hermitean operators represent observables by explicitly denying the "observability" of all operators having different mean values for those state vectors which must be equivalent.

The concept of superselection rule is relatively new and still under development. It is therefore impossible to make very definite statements about it.[28] The only point to be made here is that a standard "theorem" which purports to *derive* (2) from (1) is fallacious because it confuses measurement and preparation. A typical presentation of the argument is given by Schweber [16]: "If not all rays are realizable, then clearly no measurement can give rise to these nonrealizable states. They cannot therefore be eigenfunctions of any Hermitean operator which corresponds to an observable property of the system. To be observable a Hermitean operator must therefore satisfy certain conditions (superselection rules)." The first sentence is incontestable; indeed, if a state vector cannot be prepared at all, certainly no measurement process can do the job. The second sentence is a nonsequitur obviously based on the false premise that an $\mathscr{M}_2(\mathscr{A})$ yielding a_k is the same as $\Pi(P_{\alpha_k})$. Thus if $\Pi(P_{\alpha_k})$ is impossible, $\mathscr{M}_2(\mathscr{A})$ must be impossible. The third sentence follows from the second and is just alternative (2) above. We see therefore that confusion of measurement and preparation is here responsible for the theoreti-

[28] In our opinion alternative (1), a natural generalization of the Pauli principle, is preferable to (2) and is all that is really needed to account for physical facts of the type which suggest the existence of superselection rules, e.g. the fact that the superposition of an electron state vector and a positron state vector apparently describes no actual ensemble. If (2) were unnecessary, the word *some* would be unnecessary in P1, and P2 would not have to make the rather odd demand that $m(A)$ be real even when A is an Hermitean operator representing no observable.

cal illusion that (2) is a consequence of (1), whereas in fact (2), if needed, should be postulated independently.

Although $\mathscr{M}_1$, $\mathscr{M}_2$, and Π must never be regarded as equivalent, there are of course connections among them which we do not wish to deny. These relations may be expressed as follows: (1) $\mathscr{M}_2$ and Π are both laboratory processes the quantum theoretical description of which necessarily involves the primitive construct $\mathscr{M}_1$. (2) $\mathscr{M}_2(\mathscr{A})$, like any physical process, leaves the systems involved in *some* state; and the ensembles of these systems would have calculable density operators. In this trivial sense, all physical processes, $\mathscr{M}_2(\mathscr{A})$ included, prepare states. However, these states need not exhibit any special relation to $\mathscr{A}$; in particular, preparations effected by $\mathscr{M}_2(\mathscr{A})$ are not necessarily, nor even usually, $\Pi(P_{\alpha_k})$. To defend his use of an α_k in a complex situation where $\mathscr{M}_2(\mathscr{A})$ would never be performed, Schrödinger [15] once remarked: "A purist might challenge the use of a wave function not determined by measurement. But he would have to give up using wave functions altogether, since none has ever been determined by measurement." (3) Similarly, the physical process $\Pi(P_{\alpha_k})$ might conceivably be utilized as an $\mathscr{M}_2$, perhaps even $\mathscr{M}_2(\mathscr{A})$; but this need not be the case. Usually all that is known from $\Pi(P_{\alpha_k})$ is that, if $\mathscr{M}_2(\mathscr{A})$ were performed, the correlations thereby established would show that an $\mathscr{A}$-measurement, i.e. $\mathscr{M}_1(\mathscr{A})$, must yield a_k.

15. Summary: quantum theory of measurement. The problem of quantum measurement was introduced in section 1 in the customary way as a logical challenge to be met within the quantal framework. At issue was the fact that the explicit appearance of the term *measurement* in the postulates of quantum theory automatically confers some properties upon that concept. Yet the notion of measurement does not really belong to quantum physics in particular; indeed measurement is basic to all of physical science and presumably comes to quantum theory already endowed with characteristics inherent in its more general epistemological role. A "quantum theory of measurement" would then be a confrontation of the measurement concept in quantum theory with the idea of measurement in general in order to demonstrate the consistency of the quantum viewpoint.

However, a fundamental defect in this program gradually became apparent. The quantum measurement construct is well defined in the sense that the postulates offer clear instructions as to its use in theoretical explanations of physical processes. On the other hand, the general philosophical understanding of measurement cannot be expressed in simple mathematical terms. To demonstrate this point, we critically surveyed the major classes of proposed quantum measurement theories; invariably, the extraquantal strictures placed upon measurement were found to result in physically unwarranted "overspecifications" of its root meaning. Scrutiny of these overly narrow definitions of measurement served mainly to expose misunderstandings about the nature of quantum theory.

Eventually we recognized that this entire approach was foredoomed. Even if a grand, all-embracing mathematical definition of a general measurement process were discovered, it could not serve to establish the consistency of the quantum theoretical usage of the term measurement; for no physical process, measurement schemes included, can be described by quantum theory without the term measurement, which has therefore the logical status of an ultimate primitive, irreducible to other constructs.

This state of affairs, essentially a consequence of the latency of quantum observables, suggested the necessity of distinguishing between $\mathcal{M}_1$ and $\mathcal{M}_2$. The nature of these two measurement constructs as well as their significance for quantum measurement theory was explained in several preceding sections. However, to clarify these ideas, we shall now briefly recapitulate by describing $\mathcal{M}_1$ and $\mathcal{M}_2$ in another way, viz. by focusing upon their epistemological status. To insure direct passage through the sometimes labyrinthine halls of scientific epistemology, it is helpful to refer to a chart (Fig. 1) originated by Margenau [10].

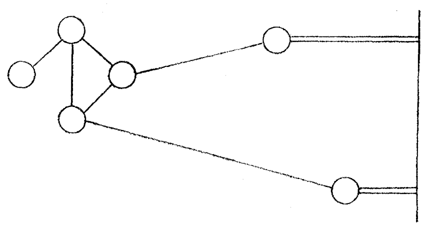

Fig. 1

A simplified legend for this "epistemological map" would make these identifications: (1) The P-plane represents uninterpreted sense impressions, those elements of experience variously called the given, the percepts, the direct observations, data, and by Margenau, the protocols. (2) The C-field is the domain of reason, of ideal models; its members (denoted by circles) are known as the categories, concepts, or constructs; and in Einstein's words, they are "free creations of the human mind" [3]. (3) A set of rational connections (single lines) among constructs forms the logical matrix of a theory. (4) Some constructs are related to direct observations at the perceptual level (P-plane) by conventions (double lines) which may be called operational definitions (Bridgman [1]), rules of correspondence (Margenau [11]), or epistemic correlations (Northrop [13]). (5) The distance of a construct from the P-plane is to be regarded as an indication of its relative abstractness, or, in a sense, its objectivity. This horizontal "scale" is of course rather vague, but it is not meaningless. For example, the construct "electric field" is obviously far to the left of "electric shock"; the sequence of concepts "entropic-derivative-of-internal-energy," "thermometric-temperature," and "hotness" evidently range from extremely far into the C-field to extremely close to the P-plane.

The general notion of measurement as a universal feature of the scientific method —what we have designated $\mathcal{M}_2$—refers to an important part of the complex of linkages between the most profound constructs and the practically self-evident ones just short of the diffuse boundary of raw, undifferentiated percepts. Measurement$_2$ is concerned directly with those constructs called observables which mediate between *mathematical* models and direct observations; the defining characteristic of any measurement_2 scheme is therefore the extraction of *numbers* from observations and their theoretically meaningful assignment to the observables. The overall purpose and pragmatic value of this procedure is fully discussed in books on the philosophy of science, but these matters are not at issue here.

With the above understanding of measurement_2 as the provider of numbers to observables, it is easy to describe in a general way, using Margenau charts, just what a "theory of measurement" would be. Consider an observable $\mathscr{A}$ which is defined constitutively by the properties of its representative A among the mathematical constructs of the theory. Suppose that an operation has been discovered the performance of which yields numbers and that these numbers can be consistently associated with $\mathscr{A}$, in some sense, as its "values." The measurement concept $\mathscr{M}_2(\mathscr{A})$ is then simply the rule of correspondence which specifies that operation (Fig. 2).

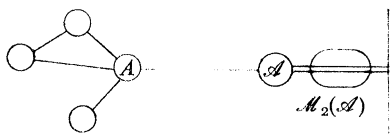

Fig. 2

A theory of measurement_2 is then simply a theoretical analysis of the operation identified as $\mathscr{M}_2(\mathscr{A})$. In other words, part of the rule of correspondence itself is explained in terms of fundamental constructs. As a result, the concept $\mathscr{M}_2(\mathscr{A})$ acquires a more complex "structure" (Fig. 3).

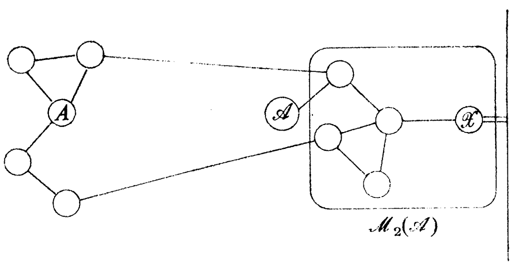

Fig. 3

On the epistemological chart, the unanalyzed double line connecting $\mathscr{A}$ to the distant P-plane is replaced by theoretical connections from $\mathscr{A}$ to an observable $\mathscr{X}$ plus a very short double line from $\mathscr{X}$ to the nearby protocols. The closeness of $\mathscr{X}$ to the P-plane signifies that, as far as physics is concerned, $\mathscr{X}$ is regarded as directly observable. In earlier sections, $\mathscr{X}$ was taken to be position, said to be measured by "looking at" the coincidence of a "spot" and a "scale marking." However, we adopted this specific identification of $\mathscr{X}$, suggested by the writings of de Broglie and Landé, only to exemplify the ultimate contact of physical theory and empirical experience; there is no reason to regard it as the sole direct observable of potential value for physics.

What has been said about measurement$_2$ thus far has been applicable to science in general. Hence problems motivated by the foregoing remarks cannot be legitimately interpreted as quantum dilemmas in particular. For example, the unanalyzed connection of an $\mathscr{X}$ to the diffuse realm of immediacy suggests the problem of infinite regression quite independently of quantum theory, as noted in section **15.** At any rate, the measurement concept $\mathscr{M}_2(\mathscr{A})$ is epistemologically the same in quantum physics as in the rest of science; and a quantum theory of $\mathscr{M}_2(\mathscr{A})$ should be of no more philosophic interest than are classical disciplines such as thermometry and photometry.

On the other hand, the measurement concept $\mathscr{M}_1$ *is* peculiar to quantum theory. To find its proper "location" on the Margenau chart, recall that the essence of measurement$_2$ is the extraction of numbers from observations and their theoretically meaningful *assignment to the observables*. Hence the description of an $\mathscr{M}_2(\mathscr{A})$ must always be given in terms of such assignments; but we have seen in previous sections that classical and quantal physics do not employ the same relation between an observable and its numerical values. Classically, an observable is said to "have" its value; quantally, the only connection is through the auxiliary measurement concept $\mathscr{M}_1(\mathscr{A})$. Accordingly, on the epistemological chart, let us replace the observable symbol ($\mathscr{A}$) by ($\mathscr{Y}(\mathscr{A})$) if $\mathscr{A}$ "has" a value and by ($\mathscr{M}_1(\mathscr{A})$) if the latent results of potential $\mathscr{M}_1(\mathscr{A})$'s represent the only connection between $\mathscr{A}$ and its values. Comparison of the classical (Fig. 4) and quantal (Fig. 5) realizations of Fig. 3 then serves to clarify the epistemological status of $\mathscr{M}_1$.

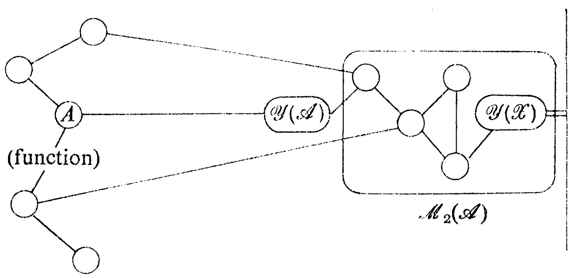

Fig. 4

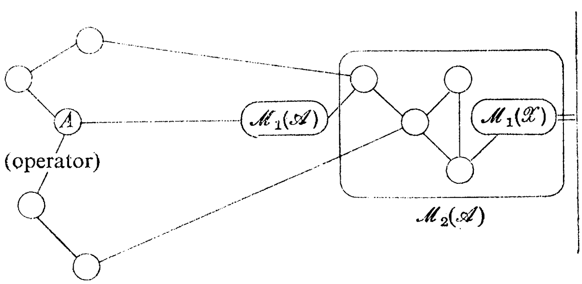

Fig. 5

These charts represent an attempt to summarize graphically the main points about quantum measurement concepts discussed in previous sections. In particular, they emphasize that $\mathscr{M}_1$ is analogous to the classical idea $\mathscr{Y}$ of "having" a value. It is meaningless to consider further analysis of either $\mathscr{Y}$ or $\mathscr{M}_1$; logically, both are ultimate primitives in their respective theories in the sense that no physical process can be described without them. There could no more be a "quantum theory of $\mathscr{M}_1$" than there could be a "classical theory of $\mathscr{Y}$"; either would be quite circular. Hence the term "quantum theory of measurement" can only refer to a theory of $\mathscr{M}_2(\mathscr{A})$, the statement of which will necessarily employ $\mathscr{M}_1$'s. Understood in this way, so-called quantum theories of measurement$_2$ are of no more or less philosophic interest than analogous classical theories of measurement$_2$ which explain the operation of calorimeters, spectrometers, etc. Indeed the rather extraordinary qualities sometimes attributed to quantum measurement derive from the various misinterpretations of quantum theory which the present work has sought to expose. Once the distinctness of $\mathscr{M}_1$ and $\mathscr{M}_2$ is recognized, the general concept of measurement $\mathscr{M}_2$ is no more mysterious in quantum physics than elsewhere.

Acknowledgment: This paper reflects numerous stimulating conversations with Professor Henry Margenau, for whose scholarly guidance I am deeply grateful.

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